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Line Graph of Friendship Graph and CTI

Albina A*, Manonmani A
Department of Mathematics, L.R.G. Govt. Arts College (W), Tirupur, India

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ABSTRACT

Topological indices are used to describe the symmetry of molecular structures and to predict features like boiling temperatures, viscosity, gyroscope radius, etc.[8] In recent work, the idea of chromatic topological indices has been explored as an coloring version of particular Zagreb indices. In this article, new indices for the line graph of friendship graphs have been developed.

Keywords: Chromatic Zagreb Indices, Friendship Graph, Irregularity indices, Line Graph.

I. INTRODUCTION

A topological index or a molecular structure descriptor is a numerical value associated with chemical structures with various physical properties. The connectivity index and its variants are used more frequently than any other topological index in QSPR and QSAR [3,5,6,7] .

J.Kok et. al. found the general formulae of chromatic Zagreb indices for certain graphs such as tree, caterpillar, etc., [4]. Let $C = \{c_1, c_2, \ldots, c_l\}$ be the proper coloring of any graph G . Since $|C| = l$, G has $l!$ minimum parameter colorings. Denote these colorings as $\phi_t(G), 1 \le t \le l!$.

The minimum and maximum chromatic Zagreb indices, as well as the related irregularity indices, are defined as follows:

$$M_i^{\phi^-}(G) = \min\{M_i^{\phi_t(G)} : 1 \le t \le l!\}, 1 \le i \le 4$$
$$M_i^{\phi^+}(G) = \max\{M_i^{\phi_t(G)} : 1 \le t \le l!\}, 1 \le i \le 4$$

II. LINE GRAPH OF A FRIENDSHIP GRAPH

Definition 1: [1]A fan graph F_n is defined as the join of the path P_n and the graph $\widetilde{K}_1$. That is $P_n + \widetilde{K}_1$.

Definition 2: [2]The line graph of a graph G denoted by $L(G)$, is the graph whose vertices are the edges of G, with two vertices of $L(G)$ are adjacent whenever the corresponding edges of G are adjacent.

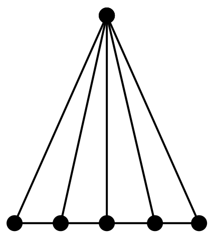

Figure 1: Fan Graph F_5.

Properties of Line Graph

- Order of $L(F_n)$ = Size of $F_n = m = 2n-1$.
- Size of $L(F_n) = \frac{n^2+5n-8}{2}$
- Chromatic number of $L(F_n) = \chi(L(F_n)) = n$.

III. MINIMUM CHROMATIC TOPOLOGICAL INDICES

A. First Zagreb Index

[4]The *first chromatic Zagreb index* of a graph G is defined as:

$$M_1^{\phi_t}(G) = \sum_{i=1}^{n} c(v_i)^2$$
$$= \sum_{j=1}^{l} \theta(c_j)\cdot j^2, c_j \in C$$

THEOREM 1:

$$M_1^{\phi^-}(L(F_n)) = \begin{cases} \dfrac{2n^3+3n^2+16n-15}{6}, & if\ n\ is\ odd \\ \dfrac{2n^3+3n^2+16n+24}{6}, & if\ n\ is\ even \end{cases}$$

Proof:

Let $e_1, e_2, \cdots, e_{2n-1}$ be the edge set of F_n which represents the vertex set of F_n such that, $e_1, e_2, \cdots, e_n$ be the edges joining the vertex of $\widetilde{K}_1$ to the vertices of $P_n$and $e_{n+1}, \cdots, e_{2n-1}$ represents the edges of P_n.

Case 1: When n is odd

Consider the ϕ^- coloring of $L(F_n)$ as follows: $c(e_1) = 1, c(e_n) = 2, c(e_i) = i+1; 2 \le i \le n-1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = 1$ and $c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-1}) = 2$.

From the above coloring we obtain that $\theta(1) = \theta(2) = \frac{n+1}{2}$ and $\theta(i) = 1; 3 \le i \le n$.

$$M_1^{\phi^-}(G) = \sum_{i=1}^{n} \theta(i)\cdot i^2$$
$$= 1\left(\frac{n+1}{2}\right) + 4\left(\frac{n+1}{2}\right) + 3^2 + \cdots + n^2$$
$$= \frac{5n+5}{2} + 1^2 + 2^2 + \cdots + n^2 - 1^2 - 2^2$$
$$= \frac{5n-5}{2} + \frac{n(n+1)(2n+1)}{6}$$
$$= \frac{2n^3+3n^2+16n-15}{6}$$

Case 2: When n is even

Consider the ϕ^- coloring of $L(F_n)$ as follows: $c(e_1) = 1$, $c(e_n) = 2$, $c(e_i) = i+1; 2 \le i \le n-1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = 1$, $c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-3}) = 2$ and $c(e_{2n-1}) = 3$.

From the above coloring we obtain that $\theta(1) = \theta(2) = \frac{n}{2}, \theta(3) = 2$ and $\theta(i) = 1; 4 \le i \le n$.

$$M_1^{\phi^-}(G) = \sum_{i=1}^{n} \theta(i)\cdot i^2$$
$$= 1\left(\frac{n}{2}\right) + 2^2\left(\frac{n}{2}\right) + 3^2(2) + 4^2 + \cdots + n^2$$
$$= \frac{5n}{2} + 18 + 1^2 + 2^2 + \cdots + n^2 - 1^2 - 2^2 - 3^2$$
$$= \frac{5n}{2} + 4 + \frac{n(n+1)(2n+1)}{6}$$
$$= \frac{2n^3+3n^2+16n+24}{6}$$

B. Second Zagreb Index

[4]The *second chromatic Zagreb index* of G is defined as:

$$M_2^{\phi_t}(G) = \sum_{i=1}^{n-1}\sum_{j=2}^{n}(c(v_i)\cdot c(v_j)), v_iv_j \in E(G)$$
$$= \sum_{1\le t,s\le l}^{t<s}(t\cdot s)\eta_{ts}$$

THEOREM 2:

$$M_2^{\phi^-}(L(F_n)) = \begin{cases} 3n^2+5n-16+\sum\limits_{3\le i<j\le n}(i\cdot j), & if\ n\ is\ odd \\ \dfrac{9n^2+19n-58}{2} + \sum\limits_{3\le i<j\le n}(i\cdot j), & if\ n \ge 6, is\ even \\ 11, & if\ n=2 \\ 73, & if\ n=4 \end{cases}$$

Proof:

Case 1: When n is odd

Consider the ϕ^- coloring of $L(F_n)$ as follows: $c(e_1) = 1, c(e_n) = 2, c(e_i) = i+1; 2 \le i \le n-1$, $c(e_{n+1}) =$

$c(e_{n+3}) = \cdots = c(e_{2n-2}) = 1$ and $c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-1}) = 2$.

From the above coloring we get, $\eta_{12} = n + 1, \eta_{1i} = \eta_{2i} = 2; 3 \leq i \leq n$ and $\eta_{ij} = 1; 3 \leq i < j \leq n$.

$$M_2^{\phi^-}(G) = \sum_{1 \leq i < j \leq n} (i \cdot j)\eta_{ij}$$
$$= 2\eta_{12} + 3\eta_{13} + \cdots + n\eta_{1n} + 6\eta_{23} + \cdots + 2n\eta_{2n} + \cdots + n(n-1)\eta_{n(n-1)}$$
$$= 2(n+1) + 3 \cdot 2 + 4 \cdot 2 + \cdots + n \cdot 2 + 2 \cdot 3 \cdot 2 + 2 \cdot 4 \cdot 2 + \cdots + 2 \cdot n \cdot 2 + 3 \cdot 4 + 3 \cdot 5 + \cdots + 3 \cdot n + 4 \cdot 5 + \cdots + \cdots + n(n-1)$$
$$= 2n + 2 + 2(1 + 2 + \cdots + n) - 6 + 4(1 + 2 + \cdots + n) - 12 + \sum_{3 \leq i < j \leq n} (i \cdot j)$$
$$= 2n - 16 + (2 + 4)\left(\frac{n(n+1)}{2}\right) + \sum_{3 \leq i < j \leq n} (i \cdot j)$$
$$= 3n^2 + 5n - 16 + \sum_{3 \leq i < j \leq n} (i \cdot j)$$

Case 2: n=2

Consider the ϕ^- coloring of $L(F_2)$ as follows: $c(e_1) = 1,\ c(e_n) = 2,\ c(e_3) = 3$. Hence $\eta_{12} = \eta_{13} = \eta_{23} = 1$.

Therefore $M_2^{\phi^-}(G) = 2\eta_{12} + 3\eta_{13} + 6\eta_{23} = 11$.

Case 3:n=4

Consider the ϕ^- coloring of $L(F_4)$ as follows: $c(e_1) = c(e_6) = 1,\ c(e_4) = c(e_5) = 2,\ c(e_2) = c(e_7) = 3$ and $c(e_4) = 4$. Hence $\eta_{12} = \eta_{13} = \eta_{23} = 3, \eta_{14} = \eta_{34} = 2$ and $\eta_{24} = 1$.

Therefore $M_2^{\phi^-}(G) = 2\eta_{12} + 3\eta_{13} + 4\eta_{14} + 6\eta_{23} + 8\eta_{24} + 12\eta_{34} = 6 + 9 + 8 + 18 + 8 + 24 = 73$.

Case 4: When n is even and n ≥ 6

Consider the ϕ^- coloring of $L(F_n)$ as follows: $c(e_1) = 1,\ c(e_n) = 2,\ c(e_i) = i + 1; 2 \leq i \leq n - 1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = 1$, $c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-3}) = 2$ and $c(e_{2n-1}) = 3$.

From the above coloring we get $\eta_{12} = n - 1,\ \eta_{13} = \eta_{23} = 3,\ \eta_{1n} = \eta_{3n} = 2,\ \eta_{2n} = 1,\ \eta_{1i} = \eta_{2i} = 2, \eta_{3i} = 1; 4 \leq i \leq n - 1$ and $\eta_{ij} = 1; 4 \leq i < j \leq n$.

$$M_2^{\phi^-}(G) = 2(n-1) + 3(3) + 2(4 + 5 + \cdots + n) + 6(3) + 4(4 + 5 + \cdots + n - 1) + 2n + 3(4 + 5 + \cdots + n - 1) + 6n + \sum_{4 \leq i < j \leq n} (i \cdot j)$$
$$= 5n - 29 + (2 + 4 + 3)\frac{n(n+1)}{2} + \sum_{4 \leq i < j \leq n} (i \cdot j)$$
$$= \frac{9n^2 + 19n - 58}{2} + \sum_{4 \leq i < j \leq n} (i \cdot j)$$

C. Irregularity Index

[4]The *chromatic irregularity index* of G is defined as:

$$M_3^{\phi_t}(G) = \sum_{i=1}^{n-1}\sum_{j=2}^{n} |c(v_i) - c(v_j)|, v_i v_j \in E(G)$$
$$= \sum_{1 \leq t, s \leq l}^{t<s} |t - s|\eta_{ts}$$

Here η_{ts} is the number of edges $e(=uv)$ in G such that $c(u) = t$ and $c(v) = s$.

THEOREM 3:

$$M_3^{\phi^-}(L(F_n)) = \begin{cases} 2n^2 - 3n + 1 + \sum_{i=1}^{n-3} \frac{i(i+1)}{2}, & \text{if } n \text{ is odd} \\ \frac{5n^2 - 11n + 12}{2} + \sum_{i=1}^{n-4} \frac{i(i+1)}{4}, & \text{if } n \text{ is even} \\ 4, \quad \text{if } n = 2 \\ 22, \quad \text{if } n = 4 \end{cases}$$

Proof:

Case 1: When n is odd

Consider the ϕ^- coloring of $L(F_n)$ as follows: $c(e_1) = 1, c(e_n) = 2, c(e_i) = i + 1; 2 \leq i \leq n - 1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = 1$ and $c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-1}) = 2$.

From the above coloring we get, $\eta_{12} = n + 1, \eta_{1i} = \eta_{2i} = 2; 3 \leq i \leq n$ and $\eta_{ij} = 1; 3 \leq i < j \leq n$.

$$M_3^{\phi^-}(G) = \sum_{1\le i<j\le n} |i-i|\eta_{ij}$$
$$= 1\eta_{12} + 2\eta_{13} + \cdots + (n-1)\eta_{1n} + 1\eta_{23} + \cdots + (n-2)\eta_{2n} + \cdots + \eta_{n(n-1)}$$
$$= (n+1) + 2\cdot 2 + 3\cdot 2 + \cdots + (n-1)\cdot 2 + 1\cdot 2 + 2\cdot 2 + \cdots + (n-2)\cdot 2 + 1 + 2 + \cdots +(n-3) + 1 + 2 + \cdots + 9n-4) + \cdots + 1 + 2 + 1$$
$$= n + 1 + 2(1+2+\cdots+n-1) - 2 + 2(1+2+\cdots+n-2+n-1) - 2n + 2 + \sum_{i=1}^{n-3}\frac{i(i+1)}{2}$$
$$= 1 - n + (2+2)\left(\frac{n(n-1)}{2}\right) + \sum_{i=1}^{n-3}\frac{i(i+1)}{2}$$
$$= 2n^2 - 3n + 1 + \sum_{i=1}^{n-3}\frac{i(i+1)}{2}$$

Case2: n=2

Consider the ϕ^- coloring of $L(F_2)$ as follows: $c(e_1) = 1,\ c(e_n) = 2,\ c(e_3) = 3$. Hence $\eta_{12} = \eta_{13} = \eta_{23} = 1$. Therefore $M_3^{\phi^-}(G) = 1\eta_{12} + 2\eta_{13} + 1\eta_{23} = 4$.

Case 3:n=4

Consider the ϕ^- coloring of $L(F_4)$ as follows: $c(e_1) = c(e_6) = 1,\ c(e_4) = c(e_5) = 2,\ c(e_2) = c(e_7) = 3$ and $c(e_4) = 4$. Hence $\eta_{12} = \eta_{13} = \eta_{23} = 3, \eta_{14} = \eta_{34} = 2$ and $\eta_{24} = 1$.

Therefore $M_2^{\phi^-}(G) = \eta_{12} + 2\eta_{13} + 3\eta_{14} + \eta_{23} + 2\eta_{24} + \eta_{34} = 22$.

Case 4: When n is even and n ≥ 6

Consider the ϕ^- coloring of $L(F_n)$ as follows: $c(e_1) = 1,\ c(e_n) = 2,\ c(e_i) = i+1; 2 \le i \le n-1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = 1$, $c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-3}) = 2$ and $c(e_{2n-1}) = 3$.

From the above coloring we get $\eta_{12} = n-1, \eta_{13} = \eta_{23} = 3,\ \eta_{1n} = \eta_{3n} = 2,\ \eta_{2n} = 1,\ \eta_{1i} = \eta_{2i} = 2, \eta_{3i} = 1; 4 \le i \le n-1$ and $\eta_{ij} = 1; 4 \le i < j \le n$.

$$M_3^{\phi^-}(G) = (n-1) + 2(3) + 2(3+4+\cdots+n-1) + 1(3) + 2(2+3+\cdots+n-3) + (n-2) + 2n + (1+2+\cdots+n-4) + 2(n-3) + \sum_{i=1}^{n-4}\frac{i(i+1)}{2}$$
$$= n + 1 + 2\frac{n(n-1)}{2} + 2\frac{(n-2)(n-1)}{2} + \frac{(n-3)(n-2)}{2} + \sum_{i=1}^{n-4}\frac{i(i+1)}{2}$$
$$= \frac{5n^2 - 11n + 12}{2} + \sum_{i=1}^{n-4}\frac{i(i+1)}{2}$$

D. Total Irregularity Index

[4]The *chromatic total irregularity index* of G is defined as:

$$M_4^{\phi_t}(G) = \frac{1}{2}\sum_{i=1}^{n-1}\sum_{j=2}^{n}|c(v_i) - c(v_j)|, v_i, v_j \in V(G)$$

Here η_{ts} is the number of edges $e(=uv)$ in G such that $c(u) = t$ and $c(v) = s$.

THEOREM 4:

$$M_4^{\phi^-}(L(F_n)) = \begin{cases} \dfrac{n(n+1)(2n-3)}{8} + \displaystyle\sum_{i=1}^{n-3}\frac{i(i+1)}{4}, & \text{if } n \text{ is odd} \\ \dfrac{n^3 + 2n^2 - 14n + 24}{8} + \displaystyle\sum_{i=1}^{n-4}\frac{i(i+1)}{4}, & \text{if } n \text{ is even} \end{cases}$$

Proof:

Case 1: When n is odd

Consider the ϕ^- coloring of $L(F_n)$ as follows: $c(e_1) = 1, c(e_n) = 2, c(e_i) = i+1; 2 \le i \le n-1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = 1$ and $c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-1}) = 2$.

From the above coloring we obtain that $\theta(1) = \theta(2) = \frac{n+1}{2}$ and $\theta(i) = 1; 3 \le i \le n$.

$$M_4^{\phi^-}(G) = \frac{1}{2}\sum_{1\le i<j\le n}|i-j|\theta(i)\theta(j)$$

$$2M_4^{\phi^-}(G) = 1\left(\frac{n+1}{2}\right)\left(\frac{n+1}{2}\right) + \left(\frac{n+1}{2}\right)(2+3+\cdots+n-1) + \left(\frac{n+1}{2}\right)(1+2+\cdots+n-2) + \sum_{i=1}^{n-3}\frac{i(i+1)}{2}$$
$$= \left(\frac{n+1}{2}\right)\left(\frac{n+1}{2} + \frac{n(n-1)}{2} - 1 + \frac{(n-2)(n-1)}{2}\right) + \sum_{i=1}^{n-3}\frac{i(i+1)}{2}$$
$$= \left(\frac{n+1}{2}\right)\left(\frac{n(2n-3)}{2}\right) + \sum_{i=1}^{n-3}\frac{i(i+1)}{2}$$
$$M_4^{\phi^-}(G) = \left(\frac{n(n+1)(2n-3)}{8}\right) + \sum_{i=1}^{n-3}\frac{i(i+1)}{4}$$

Case 2: When n is even

Consider the ϕ^- coloring of $L(F_n)$ as follows: $c(e_1) = 1,\ c(e_n) = 2,\ c(e_i) = i+1; 2 \le i \le n-1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = 1$, $c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-3}) = 2$ and $c(e_{2n-1}) = 3$.

From the above coloring we obtain that $\theta(1) = \theta(2) = \frac{n}{2}, \theta(3) = 2$ and $\theta(i) = 1; 4 \le i \le n$.

$$2M_4^{\phi^-}(G) = 1\left(\frac{n}{2}\right)\left(\frac{n}{2}\right) + 2\left(\frac{n}{2}\right)2 + \left(\frac{n}{2}\right)(3+4+\cdots+n-1) + \left(\frac{n}{2}\right)2 + \left(\frac{n}{2}\right)(2+3+\cdots+n-2) + 2(1+2+\cdots+n-3) + \sum_{i=1}^{n-4}\frac{i(i+1)}{2}$$
$$= \frac{n^3+2n^2-14n+24}{4} + \sum_{i=1}^{n-4}\frac{i(i+1)}{2}$$
$$M_4^{\phi^-}(G) = \left(\frac{n^3+2n^2-14n+24}{8}\right) + \sum_{i=1}^{n-4}\frac{i(i+1)}{4}$$

IV. MAXIMUM CHROMATIC TOPOLOGICAL INDICES

A. First Zagreb Index

THEOREM 5:

$$M_1^{\phi^+}(L(F_n)) = \begin{cases} \dfrac{8n^3-9n^2+10n-3}{6}, & \text{if } n \text{ is odd} \\ \dfrac{8n^3-9n^2-8n+18}{6}, & \text{if } n \text{ is even} \end{cases}$$

Proof:

Let $e_1, e_2, \cdots, e_{2n-1}$ be the edge set of F_n which represents the vertex set of F_n such that, $e_1, e_2, \cdots, e_n$ be the edges joining the vertex of $\widetilde{K}_1$ to the vertices of P_n and $e_{n+1}, \cdots, e_{2n-1}$ represents the edges of P_n.

Case 1: When n is odd

Consider the ϕ^+ coloring of $L(F_n)$ as follows: $c(e_1) = n, c(e_n) = n-1, c(e_i) = i-1; 2 \le i \le n-1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = n$ and $c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-1}) = n-1$.

From the above coloring we obtain that $\theta(n) = \theta(n-1) = \frac{n+1}{2}$ and $\theta(i) = 1; 1 \le i \le n-2$.

$$M_1^{\phi^+}(G) = \sum_{i=1}^{n}\theta(i)\cdot i^2$$
$$= 1^2 + 2^2 + \cdots + (n-2)^2 + (n-1)^2\left(\frac{n+1}{2}\right) + n^2\left(\frac{n+1}{2}\right)$$
$$= \frac{(n-2)(n-1)(2n-3)}{6} + \left(\frac{n+1}{2}\right)(n^2-2n+1+n^2) = \frac{8n^3-9n^2+10n-3}{6}$$

Case 2: When n is even

Consider the ϕ^+ coloring of $L(F_n)$ as follows: $c(e_1) = n,\ c(e_n) = n-1,\ c(e_i) = i-1; 2 \le i \le n-1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = n$, $c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-3}) = n-1$ and $c(e_{2n-1}) = n-2$.

From the above coloring we obtain that $\theta(n) = \theta(n-1) = \frac{n}{2}, \theta(n-2) = 2$ and $\theta(i) = 1; 1 \le i \le n-3$.

$$M_1^{\phi^+}(G) == 1^2 + 2^2 + \cdots + (n-3)^2 + (n-2)^2 2^2 + (n-1)^2\binom{n}{2} + n^2\binom{n}{2}$$
$$= \frac{(n-2)(n-1)(2n-3)}{6} + (n^2-4n+4) + \binom{n}{2}(n^2 - 2n + 1 + n^2)$$
$$= \frac{8n^3 - 9n^2 - 8n + 18}{6}$$

B. Second Zagreb Index

THEOREM 6:

$$M_2^{\phi^+}(L(F_n)) = \begin{cases} 3n^3 - 7n^2 + 6n - 2 + \sum\limits_{1\le i<j\le n-2}(i\cdot j), & \text{if } n \text{ is odd} \\ 3n^3 - 7n^2 + 3n + \sum\limits_{1\le i<j\le n-2}(i\cdot j), & \text{if } n \ge 6, \text{ is even} \\ 11, \ \text{if } n=2 \\ 93, \ \text{if } n=4 \end{cases}$$

Proof:

Case 1: When n is odd

Consider the ϕ^+ coloring of $L(F_n)$ as follows: $c(e_1) = n, c(e_n) = n-1, c(e_i) = i-1; 2 \le i \le n-1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = n$ and$c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-1}) = n-1$.

From the above coloring we obtain that $\eta_{(n-1)n} = n+1, \eta_{i(n-1)} = \eta_{in} = 2; 1 \le i \le n-2$ and $\eta_{ij} = 1; 1 \le i < j \le n-2$.

$$M_2^{\phi^+}(G) = \sum_{1\le i<j\le n}(i\cdot j)\eta_{ij}$$
$$= \sum_{3\le i<j\le n-2}(i\cdot j) + 2(n-1)(1+2+\cdots+n-2) + 2n(1+2+\cdots+n-2) + (n-1)n(n+1)$$
$$= 3n^3 - 7n^2 + 6n - 2 + \sum_{1\le i<j\le n-2}(i\cdot j)$$

Case 2: n=2

Consider the ϕ^+ coloring of $L(F_2)$ as follows: $c(e_1) = 3,\ c(e_n) = 2,\ c(e_3) = 1$. Hence $\eta_{12} = \eta_{13} = \eta_{23} = 1$. Therefore $M_2^{\phi^-}(G) = 2\eta_{12} + 3\eta_{13} + 6\eta_{23} = 11$.

Case 3:n=4

Consider the ϕ^- coloring of $L(F_4)$ as follows: $c(e_1) = c(e_6) = 4,\ c(e_4) = c(e_5) = 3,\ c(e_2) = c(e_7) = 2$ and $c(e_4) = 1$. Hence $\eta_{23} = \eta_{24} = \eta_{34} = 3, \eta_{12} = \eta_{14} = 2$ and $\eta_{13} = 1$.

Therefore $M_2^{\phi^-}(G) = 2\eta_{12} + 3\eta_{13} + 4\eta_{14} + 6\eta_{23} + 8\eta_{24} + 12\eta_{34} = 4 + 3 + 8 + 18 + 24 + 36 = 93$.

Case 4: When n is even and n ≥ 6

Consider the ϕ^+ coloring of $L(F_n)$ as follows: $c(e_1) = n,\ c(e_n) = n-1,\ c(e_i) = i-1; 2 \le i \le n-1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = n$, $c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-3}) = n-1$ and $c(e_{2n-1}) = n-2$.

From the above coloring we get $\eta_{(n-1)n} = n-1,\ \eta_{(n-2)(n-1)} = \eta_{(n-2)n} = 3,\ \eta_{1n} = \eta_{1(n-2)} = 2$, $\eta_{1(n-1)} = 1,\ \eta_{in} = \eta_{i(n-1)} = 2, \eta_{i(n-2)} = 1; 1 \le i \le n-3$ and $\eta_{ij} = 1; 1 \le i < j \le n-3$.

$$M_2^{\phi^+}(G) = \sum_{1\le i<j\le n-2}(i\cdot j) + 2(n-1)(1+2+\cdots+n-3) + 2n(1+2+\cdots+n-3) + (n-1)(n-2)(3) + (n-2)n(3) + n(n-1)(n-1)$$
$$= \sum_{1\le i<j\le n-2}(i\cdot j) + 2(n-1)(1+2+\cdots+n-2) + 2n(1+2+\cdots+n-2) + (n-1)(n-2) + (n-2)n + n(n-1)(n-1)$$
$$= 3n^3 - 7n^2 + 3n + \sum_{1\le i<j\le n-2}(i\cdot j)$$

C. Irregularity Index

THEOREM 7:

$$M_3^{\phi^+}(L(F_n)) = \begin{cases} 2n^2 - n + 1 + \sum\limits_{i=1}^{n-1}\frac{i(i+1)}{2}, & \text{if } n \text{ is odd} \\ \frac{5n^2 - 9n - 2}{2} + \sum\limits_{i=1}^{n-2}\frac{i(i+1)}{2}, & \text{if } n \text{ is even} \\ 4, \ \text{if } n=2 \\ 25, \ \text{if } n=4 \end{cases}$$

Proof:

Case 1: When n is odd

Consider the ϕ^+ coloring of $L(F_n)$ as follows: $c(e_1) = n, c(e_n) = n-1, c(e_i) = i-1; 2 \le i \le n-1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = n$ and$c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-1}) = n-1$.

From the above coloring we get, $\eta_{1n} = n+1, \eta_{1i} = \eta_{2i} = 2; 2 \le i \le n-1$ and $\eta_{ij} = 1; 2 \le i < j \le n-1$.

$$M_3^{\phi^+}(G) = 2(1+2+\cdots+n-1) + n(n+1) + \sum_{i=1}^{n-3} \frac{i(i+1)}{2}$$
$$+2(1+2+\cdots+n-2)$$
$$= (1+2+\cdots+n-1) + n(n+1) + \sum_{i=1}^{n-1} \frac{i(i+1)}{2} + (1+2+\cdots+n-2)$$
$$= \frac{(n-1)n}{2} + \frac{(n-2)(n-1)}{2} + n(n+1) + \sum_{i=1}^{n-1} \frac{i(i+1)}{2}$$
$$= 2n^2 - n + 1 + \sum_{i=1}^{n-1} \frac{i(i+1)}{2}$$

Case2: n=2

Consider the ϕ^+ coloring of $L(F_2)$ as follows: $c(e_1) = 3,\ c(e_n) = 2,\ c(e_3) = 1$. Hence $\eta_{12} = \eta_{13} = \eta_{23} = 1$. Therefore $M_3^{\phi^+}(G) = 1\eta_{12} + 2\eta_{13} + 1\eta_{23} = 4$.

Case 3:n=4

Consider the ϕ^+ coloring of $L(F_4)$ as follows: $c(e_1) = c(e_6) = 1,\ c(e_4) = c(e_5) = 4,\ c(e_2) = c(e_7) = 2$ and $c(e_4) = 3$. Hence $\eta_{12} = \eta_{14} = \eta_{24} = 3, \eta_{13} = \eta_{23} = 2$ and $\eta_{34} = 1$.

Therefore $M_2^{\phi^-}(G) = \eta_{12} + 2\eta_{13} + 3\eta_{14} + \eta_{23} + 2\eta_{24} + \eta_{34} = 25$.

Case 4: When n is even and n ≥ 6

Consider the ϕ^+ coloring of $L(F_n)$ as follows: $c(e_1) = n,\ c(e_n) = n-1,\ c(e_i) = i-1; 2 \le i \le n-1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = n$, $c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-3}) = n-1$ and $c(e_{2n-1}) = n-2$.

From the above coloring we get $\eta_{1n} = n-1,\ \eta_{12} = \eta_{2n} = 3,\ \eta_{1(n-1)} = \eta_{2(n-1)} = 2,\ \eta_{(n-1)n} = 1,\ \eta_{1i} = \eta_{in} = 2, \eta_{2i} = 1; 3 \le i \le n-2$ and $\eta_{ij} = 1; 3 \le i < j \le n-1$.

$$M_3^{\phi^+}(G) = 1(3) + 2(2+3+\cdots+n-2) + (n-1)(n-1) + (1+2+\cdots+n-4) + 2(n-3) + 3(n-2) + 2(2+3+\cdots+n-3) + 1 + \sum_{i=1}^{n-4} \frac{i(i+1)}{2}$$
$$= 3\frac{(n-2)(n-1)}{2} + (n-1)^2 + (2n-5) + \sum_{i=1}^{n-2} \frac{i(i+1)}{2}$$
$$= \frac{5n^2 - 9n - 2}{2} + \sum_{i=1}^{n-2} \frac{i(i+1)}{2}$$

D. Total Irregularity Index

THEOREM 8:

$$M_4^{\phi^-}(L(F_n)) = \begin{cases} \dfrac{3(n-1)^2(n+1)}{8} + \displaystyle\sum_{i=1}^{n-3} \frac{i(i+1)}{4}, & \text{if } n \text{ is odd} \\ \dfrac{3n^3 - n^2 - 18n + 24}{8} + \displaystyle\sum_{i=1}^{n-4} \frac{i(i+1)}{4}, & \text{if } n \text{ is even} \end{cases}$$

Proof:

Case 1: When n is odd

Consider the ϕ^+ coloring of $L(F_n)$ as follows: $c(e_1) = n, c(e_n) = n-1, c(e_i) = i-1; 2 \le i \le n-1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = n$ and$c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-1}) = n-1$.

From the above coloring we obtain that $\theta(n) = \theta(n-1) = \frac{n+1}{2}$ and $\theta(i) = 1; 1 \le i \le n-2$.

$$M_4^{\phi^+}(G) = \frac{1}{2} \sum_{1 \le i < j \le n} |i-j| \theta(i)\theta(j)$$
$$2M_4^{\phi^+}(G) = \left(\frac{n+1}{2}\right)(1+2+3+\cdots+n-2) + (n-1)\left(\frac{n+1}{2}\right)\left(\frac{n+1}{2}\right) + \sum_{i=1}^{n-3} \frac{i(i+1)}{2} + \left(\frac{n+1}{2}\right)(1+2+\cdots+n-2)$$
$$= \frac{3(n-1)^2(n+1)}{4} + \sum_{i=1}^{n-3} \frac{i(i+1)}{2}$$

$$M_4^{\phi^+}(G) = \frac{3(n-1)^2(n+1)}{8} + \sum_{i=1}^{n-3} \frac{i(i+1)}{4}$$

Case 2: When n is even

Consider the ϕ^+ coloring of $L(F_n)$ as follows: $c(e_1) = n,\ c(e_n) = n-1,\ c(e_i) = i-1; 2 \le i \le n-1$, $c(e_{n+1}) = c(e_{n+3}) = \cdots = c(e_{2n-2}) = n$, $c(e_{n+2}) = c(e_{n+4}) = \cdots = c(e_{2n-3}) = n-1$ and $c(e_{2n-1}) = n-2$.

From the above coloring we obtain that $\theta(n) = \theta(n-1) = \frac{n}{2}, \theta(n-2) = 2$ and $\theta(i) = 1; 1 \le i \le n-3$.

$$\begin{aligned} 2M_4^{\phi^+}(G) &= 1\left(\frac{n}{2}\right)(2) + \left(\frac{n}{2}\right)(2+3+\cdots+n-2) \\ &\quad + (n-1)\left(\frac{n}{2}\right)\left(\frac{n}{2}\right) \\ &\quad + 2(1+2+\cdots+n-3) \\ &\quad + (n-2)(2)\left(\frac{n}{2}\right) + \sum_{i=1}^{n-4}\frac{i(i+1)}{2} \\ &\quad + \left(\frac{n}{2}\right)(1+2+\cdots+n-3) \\ &= \frac{3n^3 - n^2 - 18n + 24}{4} \\ &\quad + \sum_{i=1}^{n-4}\frac{i(i+1)}{2} \end{aligned}$$

$$M_4^{\phi^+}(G) = \frac{3n^3 - n^2 - 18n + 24}{8} + \sum_{i=1}^{n-3} \frac{i(i+1)}{4}$$

V. CONCLUSION

In this paper we provide an outline of topological indices of line graphs of friendship graphs. The study seems to be promising for further studies as these indices can be computed for many graph classes, derived graphs and molecular structures.

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