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Dynamic Modelling for Calculating Comprehensive Stiffness of the ball bearing

Ri CholUk[1], ZhangZhunHyok[1], Ri JuYong[1], NamGung Hwan[2], Kim ChangSop[2], Sung YunSong[2], Kim JuYong[2]
[1]School of Mechanical Technology, Kim Chaek University of Technology, Pyongyang, 950003, Democratic People's Republic of Korea
[2]Pyongyang Transportation of Technology, Democratic People's Republic of Korea

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ABSTRACT

The rotor system supported by the ball bearings is widely used in various fields such as aviation, space, and machinery due to its importance. In the study of the dynamic characteristics for the ball bearings, it is important to accurately calculate the stiffness of the ball bearings. The stiffness of the ball bearings is very important in the analysis of the vibration characteristics of the rotor system. Therefore, in this paper, the method of creating a comprehensive stiffness model of the ball bearing is mentioned. In consideration of the radial clearance of the ball bearing, the radial load acting on the ball bearing was derived, and based on this, a model for calculating the Hertz contact stiffness of the ball bearing was created. Based on the load considering the radial clearance, an oil film stiffness model of the ball bearing was created under the EHL theory. Then, the comprehensive stiffness was calculated by combining Hertz contact stiffness and the oil film stiffness of the ball bearing. When the radial clearance of the ball bearing is considered, the comprehensive stiffness is larger than when the radial clearance is not taken into account, and the radial clearance of the ball bearing is an important factor that directly affects the comprehensive stiffness of the ball bearing.

Keywords : Ball Bearing, Comprehensive Stiffness, Hertz Contact Stiffness, Oil Film Stiffness, Dynamic Modeling.

I. INTRODUCTION

Rotating machinery will have various failures during the work process, causing major economic losses. A large part of it is caused by excessive vibration of the rotor part. The hazards of vibration include noise, damage to the mechanical structure, and even instability of the rotor, fracture of the shaft system, etc., causing major accidents, so the vibration performance of the rotor system has always been a problem. Rotating machinery is generally composed of a bearing and a rotor, and the bearing plays a role in supporting the rotor.

In the study of rotor vibration, the calculation of the stiffness and damping coefficient of the bearing is the key. In 1881, Hertz[1] assumed that the contact area between the rolling elements and the inner and outer rings is an ellipse, and the contact area should be distributed in a semi-ellipsoid. And using the semi-inverse solution method and through the integral transformation, the theoretical solution to the point contact and line contact problems is given, which lays the foundation for the static analysis of rolling bearings. Jones[2] established the ring control theory of high-speed ball bearings, and this theory assumes that the steel ball has only pure rolling on one raceway, which is called the controlled raceway, while there is both rolling and sliding on the other raceway, that is, the non-controlled raceway. Palmgren[3] analyzed the bearing deformation and rolling element load distribution under radial force, axial force and moment load, and established the load deformation formula for linear elastic contact problems. Houpert[4] proposed a modified formula for the elastic deformation of the linear contact when analyzing the force of the cylindrical roller, and he believed that the linear contact deformation is proportional to the 1/3 power of the contact radius. Walters[5] proposed a dynamic analysis model. The analysis model considered the four-degree-of-freedom motion equation of the steel ball and the six-degree-of-freedom motion equation of the cage, using the fourth-order Runge-Kutta method for integration. Harris[6, 8] presented a bearing model which considered the influence of the elastohydrodynamic lubrication(EHL). Walford and Stone [7] analyzed in detail the stiffness and damping of the rolling element-raceway contact pair under vibration conditions, and concluded that the stiffness of the contact pair is the Hertz deformation stiffness. Gupta [9] considered the influence of the ball movement, stress state with the interaction between the various components on the bearing dynamic characteristics in his model. McFaddenl[10] studied the influence of single and multiple geometric defects on the raceway on the vibration characteristics of the bearing. Hagiu[11] considered the film extrusion squeeze effect and elastic deformation of Hertzian contact, and studied the stiffness of high-speed angular contact ball bearing. Akrurk[12] calculated the influence of the waviness of the inner and outer rings of the ball bearing on the vibration and energy of the bearing. Venner[13] verified the performance of the EHL film under time-varying loads under pure rolling conditions. H Wu[14] analyzed the contact stress and load distribution between roller elements and raceways using Hertz theory and established the calculation method for bending moment on the bearing end faces.

Next, the creation of a comprehensive stiffness model of ball bearings has been studied by many researchers. Zhang [15] proposed a new iterative algorithm on the basis of Jones' quasi-static model and stiffness analysis model and calculated the preload and stiffness of composite bearings. Yang[16] constructed the 5-DOF stiffness matrix is constructed based on the quasi-static model of angular contact ball bearings， developed a method to analyze various spindle stiffnesses with different configurations of bearing.

However, these literatures did not consider the method of creating a comprehensive stiffness model considering the radial clearance of the ball bearing under the EHL condition. In consideration of the radial clearance of the ball bearing, the radial load acting on the ball bearing was derived, and based on this, a model for calculating the Hertz contact stiffness of the ball bearing was created. Based on the load considering the radial clearance, an oil film stiffness model of the ball bearing was created under the EHL theory.

II. Dynamic modelling of the comprehensive stiffness of the ball bearing

The calculation of the stiffness of the ball bearing is the basis for analyzing the vibration performance of the supported rotor system. This chapter comprehensively considers Hertz theory and the EHL theory, combined with related stiffness calculation principles, and proposes a comprehensive stiffness model that includes bearing elastic deformation and lubricating oil film lubrication factors under the premise of considering the ball bearing structure.

2.1 Theoretical basis

The Basic Assumptions

- Deformation between the roller and raceway follows Hertz contact theory;
- Neglecting the force concentrations produced by the ends and fillets ;
- The outer ring of the bearing keeps stationary, whereas the inner ring tilts with shaft;
- Neglecting the compression deformations between the outer raceway and housing, the inner raceway and shaft;
- The oil film thickness is much smaller than the characteristic size of the contact object;
- Consider the viscous pressure effect of lubricating oil;
- The lubrication process is isothermal and adiabatic.

Calculation principle of the comprehensive stiffness calculation of the ball bearing

~ Hertz contact stiffness calculation of the ball bearing

For the ball bearings, Hertz contact stiffness will be generated between the roller and the inner and outer rings. In this case, the connection of Hertz contact stiffness between roller and inner raceway, and Hertz contact stiffness between the roller and outer raceway can be viewed as a series connection. If k_1 is Hertz contact stiffness between the roller and the inner ring and k_2 is Hertz contact stiffness between the roller and the outer ring, according to the definition of the stiffness, Hertz contact stiffness of the ball bearing k_r is as followings:

$$\frac{1}{k_r} = \frac{1}{k_1} + \frac{1}{k_2} \tag{1}$$

Oil film stiffness calculation of the ball bearing considering EHL theory

The existence of the oil film changes the elastic deformation between the roller and the raceway. When calculating the oil film stiffness calculation, the radial elastic deformation between the inner and outer rings of the ball bearing is the radial approach amount after considering the oil film thickness. At this time, the radial load on the bearing is still F_r. According to the definition of stiffness, the oil film stiffness of the ball bearing can be obtained.

~ Comprehensive stiffness calculation of the ball bearing

The comprehensive stiffness of the ball bearing is calculated according to the definition of stiffness, after considering the connection between the oil film stiffness of the ball bearing considering the EHL theory and the Hertz contact stiffness of the ball bearing as a series connection.

2.2 Dynamic modelling of the comprehensive stiffness

~Force analysis of the ball bearing considering the radial clearance

Figure 1 is a schematic diagram of the load distribution of a ball bearing. Without considering the radial clearance and preload, under the action of the radial external load F, the force of each rolling element is different. The center o of the inner ring of the bearing moves to the point o' in the radial direction. At this time, the roller located at the bottom of the radial load action line receives the largest load and produces the largest elastic deformation.

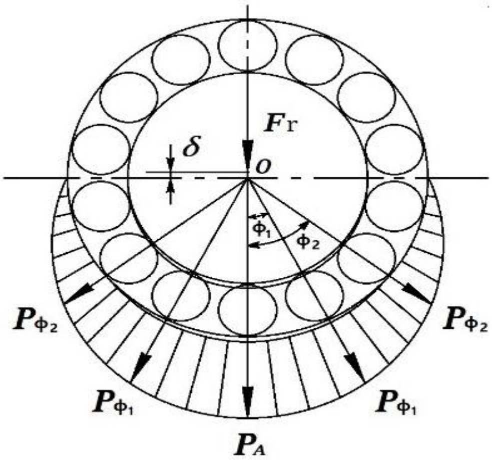

Figure 1 Radial load distribution of the ball bearing

It can be seen from Figure 1 that the maximum loaded roller is the bottom A roller.

According to the force balance:

$$F_r = P_0 + 2\sum P_{\emptyset_i} \cos \emptyset_i \tag{2}$$

Where

$\emptyset_i$ is the angle between the center line of the rolling element marked i and the action line of the radial load;

$P_{\emptyset_i}$ is the contact load borne by the rolling element whose angle with the load action line is $\emptyset_i$;

P_0 is the load on the roller on the radial load action line.

Due to the relationship between deformation and load:

$$\delta = KN^t \tag{3}$$

Therefore, the relation between contact load and deformation can be obtained:

$$\frac{P_{\emptyset_i}}{P_A} = \left(\frac{\delta_{\emptyset_i}}{\delta_A}\right)^{1/t} \tag{4}$$

According to the deformation coordination relation:

$$\delta_{\emptyset_i} = \delta_A \cos \emptyset_i \tag{5}$$

Substituting Eq.(5) into Eq. (4):

$$P_{\emptyset_i} = P_A \cos^{1/t} \emptyset_i \tag{6}$$

Substituting the above formula into the balance equation, it can be obtained that the maximum roller load applied to the inner ring is Q_A.

$$Q_A = \frac{F_r}{\cos \emptyset_i \cdot Z \cdot J_r} \tag{7}$$

where

$$J_r = \left(1 + 2\sum_{i=1}^{z}\left(1 - \frac{1}{2\Delta}(1 - \cos\emptyset_i)\right)^{1/t} \cos\emptyset_i\right)/z$$

(Considering the radial clearance)

The linear load density of the contact area between the ball and the inner ring is:

$$q_A = \frac{F_r}{J_r \cdot Z \cdot l \cdot \cos \emptyset_i} \tag{8}$$

~ Calculation of Hertz elastic deformation of the ball bearing considering the radial clearance

According to the Hertz formula of point contact elastic deformation, the contact deformation between the roller and the inner and outer raceways can be obtained respectively:

Contact deformation between roller and inner raceway is:

$$\delta_1 = 7.46X10^{-4}\frac{2k_1}{\pi m_a}\sqrt[3]{\left(\frac{F_r}{z}\right)^2 \Sigma\rho} \tag{9}$$

Contact deformation between roller and outer raceway is:

$$\delta_2 = 7.46X10^{-4}\frac{2k_2}{\pi m_a}\sqrt[3]{\left(\frac{F_r}{z}\right)^2 \Sigma\rho} \tag{10}$$

Where

$$I = 7.46X10^{-4}\frac{2k_1}{\pi m_a}\sqrt[3]{\left(\frac{1}{z}\right)^2 \Sigma\rho}$$

$$J = 7.46X10^{-4}\frac{2k_2}{\pi m_a}\sqrt[3]{\left(\frac{1}{z}\right)^2 \Sigma\rho}$$

According to Hertz theory, the total contact deformation between roller and inner and outer raceway can be deduced as:

$$\delta = \delta_1 + \delta_2 = (I + J)F_r^{2/3} \tag{11}$$

˜ Calculation of the oil film thickness of ball bearing considering radial clearance

Based on the Hamrock-Dowson oil film thickness calculation formula, the oil film thickness between the roller and the inner and outer raceways considering radial clearance can be deduced as:

$$h_{min1} = 0.44\alpha^{0.49}(\eta_0 n)^{0.7} r^{0.47}(R_1 + r)^{0.7} E^{-0.03} Z^{0.07} F_r^{\ -0.07}\left[(1-\gamma)^{1.15}(1+\gamma)^{0.7}\left(1-e_1^{0.7k_1}\right)\right] \quad (12)$$

$$h_{min2} = 0.44\alpha^{0.49}(\eta_0 n)^{0.7} r^{0.47}(R_1 + r)^{0.7} E^{-0.03} Z^{0.07} F_r^{\ -0.07}\left[(1+\gamma)^{1.15}(1-\gamma)^{0.7}\left(1-e_2^{0.7k_2}\right)\right] \quad (13)$$

$$\begin{aligned} h &= h_{min1} + h_{min2} \\ &= 0.44\alpha^{0.49}(\eta_0 n)^{0.7} r^{0.47}(R_1 + r)^{0.7} E^{-0.03} Z^{0.07} F_r^{\ -0.07}\left[(1-\gamma)^{1.15}(1+\gamma)^{0.7}\left(1-e_1^{0.7k_1}\right)\right] \\ &+ 0.44\alpha^{0.49}(\eta_0 n)^{0.7} r^{0.47}(R_1 + r)^{0.7} E^{-0.03} Z^{0.07} F_r^{\ -0.07}\left[(1+\gamma)^{1.15}(1-\gamma)^{0.7}\left(1-e_2^{0.7k_2}\right)\right] \\ &= 0.44\alpha^{0.49}(\eta_0 n)^{0.7} r^{0.47}(R_1 + r)^{0.7} E^{-0.03} Z^{0.07} F_r^{\ -0.07}\left[(1-\gamma)^{1.15}(1+\gamma)^{0.7}\left(1-e_1^{0.7k_1}\right) \right. \\ &\left. + (1+\gamma)^{1.15}(1-\gamma)^{0.7}\left(1-e_2^{0.7k_2}\right)\right] \end{aligned}$$

$$A = 0.44\alpha^{0.49}(\eta_0 n)^{0.7} r^{0.47}(R_1 + r)^{0.7} E^{-0.03} Z^{0.07}\left[(1-\gamma)^{1.15}(1+\gamma)^{0.7}\left(1-e_1^{0.7k_1}\right) + (1+\gamma)^{1.15}(1-\gamma)^{0.7}\left(1-e_2^{0.7k_2}\right)\right]$$

$$h = AF_r^{\ -0.07} \quad (14)$$

˜Comprehensive stiffness calculation of ball bearing

The stiffness of a rolling bearing refers to the ability of the rolling bearing to resist elastic deformation in the radial direction when it is subjected to a radial load.

According to the above analysis, under the action of radial load F_r, the oil film stiffness of the ball bearing is as followings:

$$k_{oil} = \lim_{\substack{\Delta F_r \to 0 \\ \Delta h \to 0}} \frac{\Delta F_r}{\Delta h} = \left(\frac{dh}{dF_r}\right)^{-1} = \left(0.07AF_r^{\ -1.07}\right)^{-1} \quad (15)$$

And the Hertz contact stiffness of the ball bearing is as followings:

$$k_{Hertz} = \lim_{\substack{\Delta F_r \to 0 \\ \Delta \delta \to 0}} \frac{\Delta F_r}{\Delta \delta} = \left(\frac{d\delta}{dF_r}\right)^{-1} = \left(\frac{2}{3}(I+J)F_r^{-1/3}\right)^{-1} \quad (16)$$

According to stiffness definition, the comprehensive stiffness of the ball bearing can be deduced as:

$$k = \left(\frac{1}{k_{oil}} + \frac{1}{k_{Hertz}}\right)^{-1} = \left(0.07AF_r^{\ -1.07} + \frac{2}{3}(I+J)F_r^{-1/3}\right)^{-1} \quad (17)$$

III. CONCLUSION

In this paper, based on Hertz contact theory and the EHL theory, we investigated the method of creating a comprehensive stiffness model of the ball bearing considering the radial clearance.

Firstly, the radial load acting on the ball bearing was derived by considering of the radial clearance of the ball bearing, and based on this, a model for calculating the Hertz contact stiffness of the ball bearing was created. Next, based on the load considering the radial clearance, an oil film stiffness model of the ball bearing was created under the EHL theory. Finally, according to the stiffness calculation theory, the comprehensive stiffness of the ball bearing was calculated by combining Hertz contact stiffness and the oil film stiffness of the ball bearing.

IV. REFERENCES

[1]. Hertz H. On the contact of elastic solidsJ]. Z. Reine Angew. Mathematik, 1881, 92: 156-171.

[2]. Jones A B. Ball motion and sliding friction in ball bearingsJ]. Journal of Basic Engineering, 1959, 81(1):1-12.

[3]. Palmgren A. Ball and roller bearing engineeringJ]. Philadelphia: SKF Industries Inc, 1959.
[4]. Houpert L. A Uniform analytical approach for ball and roller bearings calculations. Journal of Tribology. 1997, 199(4): 851-858
[5]. Walters, C. T. The Dynamics of ball bearingsJ]. Journal of Tribology, 1971, 93(1):1-10.
[6]. Harris T A, Mindel M H. Rolling element bearing dynamicsJ]. Wear, 1973, 23(3): 311-337.
[7]. Walford T L H, Stone B J. The measurement of the radial stiffness of rolling element bearings under oscillating conditionsJ]. Journal of Mechanical Engineering Science, 1980, 22(4): 175-181.
[8]. Harris T A. Rolling Bearing Analysis, John Wiley and Sons, Inc, New York, 1984.
[9]. Gupta P K. Advanced dynamics of rolling elements, Springer Verlag, Inc, New York, 1984.
[10]. McFadden P D, Smith J D. Model for the vibration produced by a single point defect in a rolling element bearingJ]. Journal of sound and vibration, 1984, 96(1): 69-82.
[11]. Hagiu G D, Gafitanu M D. Dynamic characteristics of high speed angular contact ball bearingsJ]. Wear, 1997, 211(1): 22-29.
[12]. Akturk N. The effect of waviness on vibrations associated witli ball bearings. Journal of Tribology. 1999, 121(4), 667-677.
[13]. Venner C H, Wijnant Y H. Validation of EHL contact predictions under time varying loadJ]. Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology, 2005, 219(4): 249-261.
[14]. Wu H, Sang S J, An Q. A new vibration model for a bearing-rotor system considering a bearing structureJ]. Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, 2011, 225(1): 1-11.
[15]. Zhang J, Fang B, Hong J, Wan S, and Zhu Y. A general model for preload calculation and stiffness analysis for combined angular contact ball bearingsJ]. Journal of Sound and Vibration, 2017, 411: 435-449.
[16]. Yang Z, Chen H, Yu T. Effects of rolling bearing configuration on stiffness of machine tool spindleJ]. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 2018, 232(5): 775-785.

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