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Dynamic Modelling for Calculating Comprehensive Stiffness of the Taper Roller Bearing

Ri CholUk[1], NamGung Huan[2], O RyongSik[3], Kim ChangSop[2], Han JongMin[1], Sung YunSong[2], KimJuYong[2]

[1]School of Mechanical Technology, Kim Chaek University of Technology, Pyongyang, 950003, Democratic People's Republic of Korea

[2]Pyongyang Transportation of technology, Democratic People's Republic of Korea

[3]Pyongyang University of Mechanical Engineering, Democratic People's Republic of Korea

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ABSTRACT

The rotor system supported by the taper roller bearings is widely used in various fields such as aviation, space, and machinery due to its importance. In the study of the dynamic characteristics for the taper roller bearings, it is important to accurately calculate the stiffness of the taper roller bearings. The stiffness of the taper roller bearings is very important in the analysis of the vibration characteristics of the rotor system. Therefore, in this paper, the method of creating a comprehensive stiffness model of the taper roller bearing is mentioned. In consideration of the radial clearance of the taper roller bearing, the radial load acting on the taper roller bearing was derived, and based on this, a model for calculating the Hertz contact stiffness of the taper roller bearing was created. Based on the load considering the radial clearance, an oil film stiffness model of the taper roller bearing was created under the EHL theory. Then, the comprehensive stiffness was calculated by combining Hertz contact stiffness and the oil film stiffness of the taper roller bearing. When the radial clearance of the taper roller bearing is considered, the comprehensive stiffness is larger than when the radial clearance is not taken into account, and the radial clearance of the taper roller bearing is an important factor that directly affects the comprehensive stiffness of the taper roller bearing.

Keywords : Taper roller bearing; Comprehensive stiffness; Hertz contact stiffness; Oil film stiffness; Dynamic modeling.

I. INTRODUCTION

Rotating machinery will have various failures during the work process, causing major economic losses. A large part of it is caused by excessive vibration of the rotor part. The hazards of vibration include noise, damage to the mechanical structure, and even instability of the rotor, fracture of the shaft system, etc., causing major accidents, so the vibration performance of the rotor system has always been a problem. Rotating machinery is generally composed of a bearing and a rotor, and the bearing plays a role in supporting the rotor.

In the study of rotor vibration, the calculation of the stiffness and damping coefficient of the bearing is the key. In 1881, Hertz[1] assumed that the contact area between the rolling elements and the inner and outer rings is an ellipse, and the contact area should be distributed in a semi-ellipsoid. And using the semi-inverse solution method and through the integral transformation, the theoretical solution to the point contact and line contact problems is given, which lays the foundation for the static analysis of rolling bearings. Jones[2] established the ring control theory of high-speed ball bearings, and this theory assumes that the steel ball has only pure rolling on one raceway, which is called the controlled raceway, while there is both rolling and sliding on the other raceway, that is, the non-controlled raceway. Palmgren[3] analyzed the bearing deformation and rolling element load distribution under radial force, axial force and moment load, and established the load deformation formula for linear elastic contact problems. Houpert[4] proposed a modified formula for the elastic deformation of the linear contact when analyzing the force of the cylindrical roller, and he believed that the linear contact deformation is proportional to the 1/3 power of the contact radius. Walters[5] proposed a dynamic analysis model. The analysis model considered the four-degree-of-freedom motion equation of the steel ball and the six-degree-of-freedom motion equation of the cage, using the fourth-order Runge-Kutta method for integration. Harris[6, 8] presented a bearing model which considered the influence of the elastohydrodynamic lubrication(EHL). Walford and Stone [7] analyzed in detail the stiffness and damping of the rolling element-raceway contact pair under vibration conditions, and concluded that the stiffness of the contact pair is the Hertz deformation stiffness. Gupta [9] considered the influence of the ball movement, stress state with the interaction between the various components on the bearing dynamic characteristics in his model. McFaddenl[10] studied the influence of single and multiple geometric defects on the raceway on the vibration characteristics of the bearing. Hagiu[11] considered the film extrusion squeeze effect and elastic deformation of Hertzian contact, and studied the stiffness of high-speed angular contact ball bearing. Akrurk[12] calculated the influence of the waviness of the inner and outer rings of the taper roller bearing on the vibration and energy of the bearing. Venner[13] verified the performance of the EHL film under time-varying loads under pure rolling conditions. H Wu[14] analyzed the contact stress and load distribution between roller elements and raceways using Hertz theory and established the calculation method for bending moment on the bearing end faces.

Next, the creation of a comprehensive stiffness model of ball bearings has been studied by many researchers. Zhang [15] proposed a new iterative algorithm on the basis of Jones' quasi-static model and stiffness analysis model and calculated the preload and stiffness of composite bearings. Yang[16] constructed the 5-DOF stiffness matrix is constructed based on the quasi-static model of angular contact ball bearings developed a method to analyze various spindle stiffnesses with different configurations of bearing.

However, these literatures did not consider the method of creating a comprehensive stiffness model considering the radial clearance of the taper roller bearing under the EHL condition. In consideration of the radial clearance of the taper roller bearing, the radial load acting on the taper roller bearing was derived, and based on this, a model for calculating the Hertz contact stiffness of the taper roller bearing was created. Based on the load considering the radial clearance, an oil film stiffness model of the taper roller bearing was created under the EHL theory.

II. Dynamic modelling of the comprehensive stiffness of the taper roller bearing

The calculation of the stiffness of the taper roller bearing is the basis for analyzing the vibration performance of the supported rotor system. This chapter comprehensively considers Hertz theory and the EHL theory, combined with related stiffness calculation principles, and proposes a comprehensive stiffness model that includes bearing elastic deformation and lubricating oil film lubrication factors under the premise of considering the taper roller bearing structure.

2.1 Theoretical basis

The Basic Assumptions

~Deformation between the roller and raceway follows Hertz contact theory;

~Neglecting the force concentrations produced by the ends and fillets；

~The outer ring of the bearing keeps stationary, whereas the inner ring tilts with shaft;

~Neglecting the compression deformations between the outer raceway and housing, the inner raceway and shaft;

~The oil film thickness is much smaller than the characteristic size of the contact object;

~Consider the viscous pressure effect of lubricating oil;

~The lubrication process is isothermal and adiabatic.

Calculation principle of the comprehensive stiffness calculation of the taper roller bearing

~ Hertz contact stiffness calculation of the taper roller bearing

For the taper roller bearings, Hertz contact stiffness will be generated between the roller and the inner and outer rings. In this case, the connection of Hertz contact stiffness between roller and inner raceway, and Hertz contact stiffness between the roller and outer raceway can be viewed as a series connection. If k_1 is Hertz contact stiffness between the roller and the inner ring and k_2 is Hertz contact stiffness between the roller and the outer ring, according to the definition of the stiffness, Hertz contact stiffness of the taper roller bearing k_r is as followings:

$$\frac{1}{k_r} = \frac{1}{k_1} + \frac{1}{k_2} \tag{1}$$

~Oil film stiffness calculation of the taper roller bearing considering EHL theory

The existence of the oil film changes the elastic deformation between the roller and the raceway. When calculating the oil film stiffness calculation, the radial elastic deformation between the inner and outer rings of the taper roller bearing is the radial approach amount after considering the oil film thickness. At this time, the radial load on the bearing is still F_r. According to the definition of stiffness, the oil film stiffness of the taper roller bearing can be obtained.

~ Comprehensive stiffness calculation of the taper roller bearing

The comprehensive stiffness of the taper roller bearing is calculated according to the definition of stiffness, after considering the connection between the oil film stiffness of the taper roller bearing considering the EHL theory and the Hertz contact stiffness of the taper roller bearing as a series connection.

2.2 Dynamic modelling of the comprehensive stiffness

~Force analysis of the taper roller bearing considering the radial clearance

Figure 2 is a schematic diagram of the load distribution of a tapered bearing. Suppose the contact angle of the inner raceway is α, the half cone angle of the tapered roller is β, and the angle between adjacent rollers is ∅. When radial clearance and preload are not considered, under the action of radial external load F_r, the force of each rolling element is different, and the center 0 of the inner ring of the bearing moves to 0′ point in the radial direction. At this time, the rollers located at the bottom of the radial load action line receive the largest load and produce the largest elastic deformation. Because the structure of tapered rollers is different from that of cylindrical rollers, the load distribution of tapered roller bearings is also different from that of cylindrical roller bearings.

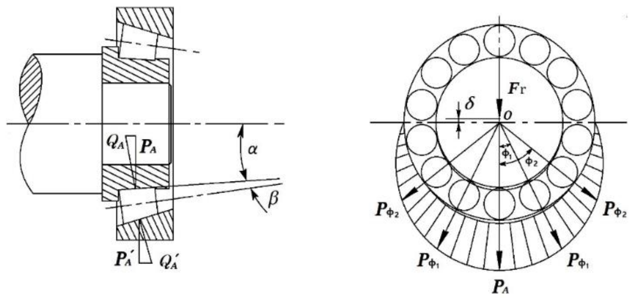

Figure 1. Radial load distribution of the taper roller bearing

It can be seen from Figure 1 that the maximum loaded roller is the bottom A roller, and the normal load acting on A by the inner ring of the bearing is Q_A: the radial component force is P_A.

According to the force balance:

$$F_r = P_A + 2\sum P_\emptyset \cos\emptyset \tag{2}$$

According to the force analysis:

$$Q_A = \frac{P_A}{\cos\alpha}$$

$$Q_B = \frac{P_{\emptyset 1}}{\cos\alpha} \text{(roller A adjacent to roller } \emptyset_1\text{)} \tag{3}$$

$$F_r = \cos\alpha\left(Q_A + 2\sum Q_\emptyset \cos\emptyset\right) \tag{4}$$

Due to the relationship between deformation and load:

$$\delta = KN^t \tag{5}$$

Therefore, the relation between contact load and deformation can be obtained:

$$\frac{P_\emptyset}{P_A} = \left(\frac{\delta_\emptyset}{\delta_A}\right)^{1/t} \tag{6}$$

According to the deformation coordination relation:

$$\delta_\emptyset = \delta_A \cos\emptyset \tag{7}$$

Substituting Eq.(7) into Eq. (6):

$$P_\emptyset = P_A \cos^{1/t}\emptyset \tag{8}$$

Substituting the above formula into the balance equation, it can be obtained that the maximum roller load applied to the inner ring is Q_A.

$$Q_A = \frac{F_r}{\cos\alpha \cdot Z \cdot J_r} \tag{9}$$

where

$$J_r = \left(1 + 2\sum_{i=1}^{z}\left(1 - \frac{1}{2\Delta}(1 - \cos\varphi_i)\right)^{1/t} \cos\varphi_i\right)/z$$

Since the load is evenly distributed along the busbar, the linear load density of the contact area between the tapered roller and the inner ring is:

$$q_A = \frac{F_r}{J_r \cdot Z \cdot l \cdot \cos\alpha} \tag{10}$$

It can be obtained that the load that the outer ring of the bearing acts vertically on the A roller busbar is:

$$Q'_A = \frac{F_r}{J_r \cdot \cos(\alpha + 2\beta) \cdot Z} \tag{11}$$

The linear load density is:

$$q'_A = \frac{F_r}{J_r \cdot Z \cdot l\, \cos(\alpha + 2\beta)} \tag{12}$$

For taper roller bearings, the centrifugal force of the rollers will change the load distribution between the outer raceway and the inner ring rib, as shown in Figure 3.

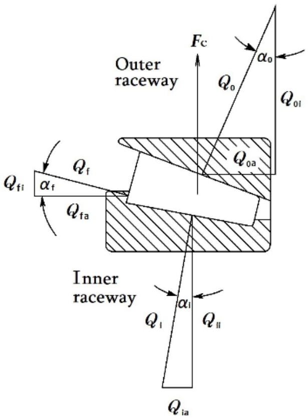

Figure 2 Centrifugal force of the taper roller

According to the equilibrium conditions:

$$Q_{ia} + Q_{fa} - Q_{oa} = 0 \tag{13}$$

$$Q_{ir} - Q_{fr} + F_C - Q_{oa} = 0 \tag{14}$$

$$Q_{ia} + Q_f \sin\alpha_f - Q_o \sin\alpha_o = 0 \tag{15}$$

$$Q_{ia} \cot\alpha_i + Q_f \cos\alpha_f - Q_o \cos\alpha_o = 0 \tag{16}$$

Solving Eq. (13) and Eq. (14) simultaneously, is as followings:

$$Q_o = \frac{Q_{ia}(\cot\alpha_i \sin\alpha_f + \cos\alpha_f) + F_c \sin\alpha_f}{\sin(\alpha_o + \alpha_f)} \tag{17}$$

$$Q_f = \frac{Q_{ia}(\cot\alpha_i \sin\alpha_o - \cos\alpha_f) + F_c \sin\alpha_o}{\sin(\alpha_o + \alpha_f)} \tag{18}$$

~ Contact stiffness calculation of the taper roller bearing considering the radial clearance

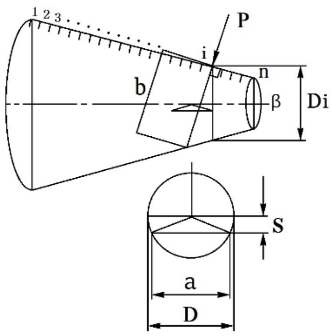

Figure 3 Curvature radius of the taper roller

Different from the property of equal radius of curvature on the generatrix of cylindrical rollers, the radius of curvature along the direction of the conical generatrix changes gradually, and the section perpendicular to the generatrix is an ellipse. Divide n equal parts along the direction of the conical generatrix, as shown in Figure 4. Let the diameter of the section of the cone passing through the i th point on the generatrix and parallel to the end face of the cone be D_i, and P is the force of the ferrule acting on the i th point of the roller and perpendicular to the generatrix. The half cone angle of the roll is β. The radius of curvature of the tapered rollers and the inner and outer raceways are calculated separately below.

The major axis of the ellipse section at point i is:

$$b'' = (D_i \cos\beta + D_i \sin\beta \tan 2\beta) = D_i(\cos\beta + \sin\beta \tan 2\beta) \tag{19}$$

The short axis of the section at point i is:

$$\alpha'' = 2\sqrt{\left(\frac{1}{2}D\right)^2 - s^2} \tag{20}$$

where s is the distance between the minor axis of the ellipse section at point i and the center of the conic section; D is the diameter of the circle passing through the minor axis of the ellipse section at the *i* th point and parallel to the end face of the roller.

$$s = \frac{1}{2}\frac{b''}{\cos\beta} - \frac{1}{2}D = \frac{1}{2}D_i\left(\tan\beta \tan 2\beta - \sin^2\beta - 2\frac{\sin^4\beta}{\cos 2\beta}\right)$$

$$D = D_i + b'' \cdot \sin\beta \cdot \tan\beta = D_i\left(1 + \sin^2\beta + 2\frac{\sin^4\beta}{\cos 2\beta}\right)$$

$$\alpha'' = D_i\sqrt{(1 + \tan\beta \tan 2\beta)\left(1 - \frac{2\sin^2\beta}{\cos 2\beta} + 2\sin^2\beta + 4\frac{\sin^4\beta}{\cos 2\beta}\right)}$$

Since β is small:

$$\frac{2\sin^2\beta}{\cos 2\beta} \approx 2\sin^2\beta,\ \sin^4\beta \to 0$$

$$\alpha'' \approx D_i\sqrt{(1 + \tan\beta \tan 2\beta)} \tag{21}$$

Get the radius of curvature of the tapered roller at point *i*.

$$r = \frac{\left(\frac{1}{2}a\right)^2}{\frac{1}{2}b} = \frac{\frac{1}{4}D_i^2(1+\tan\beta\tan 2\beta)}{\frac{1}{2}D_i(\cos\beta+\sin\beta\tan 2\beta)} = \frac{1}{2}\frac{1+\tan\beta\tan 2\beta}{\cos\beta+\sin\beta\tan 2\beta}D_i \tag{22}$$

The raceway in contact with the roller is also conical, the contact surface between the inner raceway and the roller is a convex surface, and the half angle of the top angle of the inner raceway is α. The radius of curvature of the inner raceway at point *i* is R.

Where D_2 The diameter of the section of the inner raceway parallel to the end face of the inner raceway at point *i*.

$$R_1 = \frac{1}{2}\frac{1+\tan\alpha\tan 2\alpha}{\cos\alpha+\sin\alpha\tan 2\alpha}D_2 \tag{23}$$

The contact surface between the outer raceway and the roller is concave, the half angle of the top angle of the outer raceway is α+2β, and the radius of curvature of the outer raceway at point *i* is:

$$R_2 = \frac{1}{2}\frac{1+\tan(\alpha+2\beta)\tan 2(\alpha+2\beta)}{\cos(\alpha+2\beta)+\sin(\alpha+2\beta)\tan 2(\alpha+2\beta)}D_3 \tag{24}$$

Where D_3 is the diameter of the cross-section of the outer raceway at point i parallel to the end face of the outer raceway.

According to the geometric relations:

$$D_2 = \frac{D_i}{\sin\beta}\sin\alpha,\ D_3 = \frac{D_i}{\sin\beta}\sin(\alpha+2\beta)$$

Therefore, the radius of curvature r, R_1, R_2 of the taper roller and the inner and outer rings can be simplified as:

$$r = A_1 D_i,\ R_1 = A_2 D_i,\ R_2 = A_3 D_i \tag{25}$$

Where

$$A_1 = \frac{1}{2}\frac{1+\tan\beta\tan 2\beta}{\cos\beta+\sin\beta\tan 2\beta},\ A_2 = \frac{1}{2}\frac{1+\tan\alpha\tan 2\alpha}{\cos\alpha+\sin\alpha\tan 2\alpha}\frac{\sin\alpha}{\sin\beta}$$

$$A_3 = \frac{1}{2}\frac{1+\tan(\alpha+2\beta)\tan 2(\alpha+2\beta)}{\cos(\alpha+2\beta)+\sin(\alpha+2\beta)\tan 2(\alpha+2\beta)}\frac{\sin(\alpha+2\beta)}{\sin\beta}$$

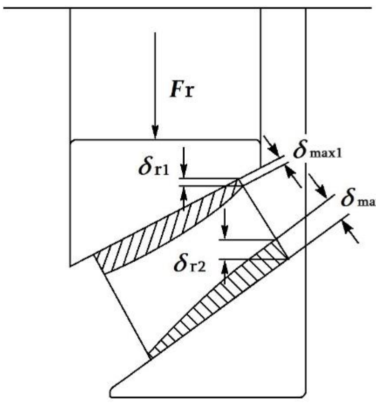

Figure 4 Approach amount between raceway

Substitute the obtained load at the largest roller and the radius of curvature r, R_1 and R_3into the basic formula of elastic deformation:

Suppose $D_i = 2\sin\beta \cdot l_i$, $l_i \in \left(\frac{D_1}{2\sin\beta}, l + \frac{D_1}{2\sin\beta}\right)$.

Where l_i -Point i is bus position.

Elastic approach between roller and inner raceway:

$$\delta_1 = \frac{2.60F_r}{ZlE\cos\alpha}\left(ln l_i + ln\frac{ZLE'(A_1+A_2)\cos\alpha\sin\beta}{F_r} + 0.58\right) \tag{26}$$

The functional relation between l_i and δ_1 can be expressed as followings:

$$\delta_1 = U_1 \cdot ln l_i + V_1$$

Where $U_1 = \frac{2.60F_r}{ZLE'\cos\alpha} > 0$

Elastic approach between roller and outer raceway:

$$\delta_2 = \frac{8.16F_r}{ZLE'\cos(\alpha+2\beta)}\left(ln l_i + \frac{1}{2}ln\frac{F_r A_1 A_3 \sin\beta}{ZLE(A_3-A_1)\cos(\alpha+\beta)} + 0.51\right) \tag{27}$$

The functional relation between l_i and δ_2 can be expressed as followings:

$$\delta_2 = U_2 \cdot ln l_i + V_2$$

Where $U_2 = \frac{8.16F_r}{ZLE\cos(\alpha+2\beta)} > 0$

~ Calculation of the oil film thickness of taper roller bearing considering radial clearance

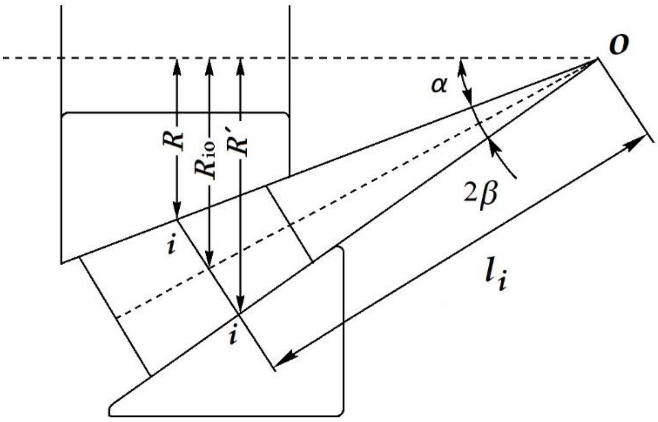

Figure 5 Motion diagram of the taper roller bearing

As shown in Figure 5, the velocity at a distance of l_i from the cone vertex 0 on the conic generatrix is $U_1 = U_2 = \omega R$.
Since $R_i = l_i \sin\alpha$:
The rolling speed of the inner ring contact pair is:
$U = \omega l_i \sin\alpha$
The rolling speed of the outer ring contact pair is:
$U' = \frac{\omega l_i \sin\alpha}{2}$
The unit load of tapered rollers on the inner and outer rings is:
$q_A = \frac{F_r}{J_r \cdot Z \cdot l \cdot \cos\alpha}$
$q_A' = \frac{F_r}{J_r \cdot Z \cdot l\, \cos(\alpha+2\beta)}$
The equivalent radius of the inner and outer rings are:
$\rho_1 = 2\sin\beta l_i \frac{A_1 A_2}{A_1+A_2}$, $\rho_2 = 2\sin\beta l_i \frac{A_1 A_3}{A_3-A_1}$

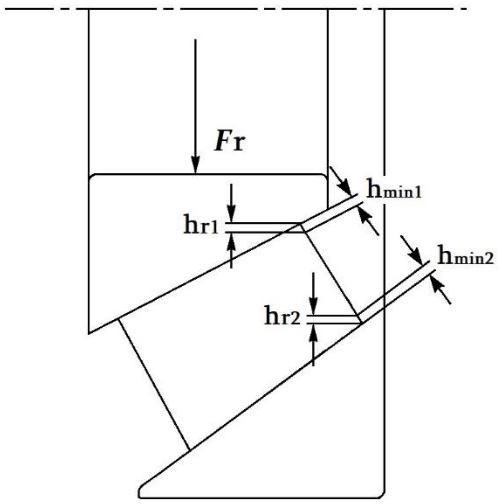

Figure 6 Oil film thickness of the biggest loaded roller

Substituting the above parameters into the minimum oil film thickness formula, the distribution of the oil film thickness at the roller with the largest load along the busbar can be obtained:

$$h_{min1} = 0.73\alpha^{0.54}E^{-0.03}(\eta_0 \omega \sin\alpha)^{0.7}\left(\frac{A_1 A_2 \sin\beta}{A_1+A_2}\right)^{0.43}\left(\frac{F_r}{ZL\cos\alpha}\right)^{-0.13} l_i^{1.13} \quad (28)$$

$$h_{min2} = 0.45\alpha^{0.54}E^{-0.03}(\eta_0 \omega \sin\alpha)^{0.7}\left(\frac{A_1 A_3 \sin\beta}{A_3-A_1}\right)^{0.43}\left(\frac{F_r}{ZL\cos(\alpha+2\beta)}\right)^{-0.13} l_i^{1.13} \quad (29)$$

~Comprehensive stiffness calculation of taper roller bearing

δ_r is the radial elastic deformation at the end face of the tapered roller, as shown in Figure 4, it represents the elastic approach between the inner and outer rings of the tapered roller bearing.

$$\delta_r = \delta_{r1} + \delta_{r2} \quad (30)$$

where

$\delta_{r1} = \frac{2.60 F_r \cos(\alpha+\beta)}{ZLE' \cos\alpha}\left(-\ln F_r + \ln\frac{D_1(A_1+A_2)\cos\alpha\sin\beta}{2\sin\beta} + 0.58\right)$

$\delta_{r2} = \frac{8.16 F_r \cos(\alpha+\beta)}{ZLE' \cos(\alpha+2\beta)}\left(\ln F_r + \frac{1}{2}\ln\frac{D_1 A_1 A_3 \sin\beta}{2ZLE'(A_3-A_1)\sin\beta\cos(\alpha+\beta)} + 0.51\right)$

δ_r can be expressed in the form $\delta_r = mF_r \cdot \ln F_r + nF_r$.
where

$m = -\frac{2.60 F_r \cos(\alpha+\beta)}{ZLE' \cos\alpha} - \frac{8.16\cos(\alpha+\beta)}{ZLE' \cos(\alpha+2\beta)}$

$$n = \frac{2.60F_r\cos(\alpha+\beta)}{ZLE'\cos\alpha}\left[ln\frac{D_1ZLE'(A_1+A_2)\cos\alpha}{2}+0.58\right]-\frac{8.16\cos(\alpha+\beta)}{ZLE'\cos(\alpha+2\beta)}\left(\frac{1}{2}ln\frac{D_1A_1A_3}{2ZLE'(A_3-A_1)\cos(\alpha+\beta)}+0.51\right)$$

h_r is the radial oil film thickness between the inner and outer rings of the bearing at the upper end of the roller and the roller, as shown in Figure 6.

$$h_r = h_{r1} + h_{r2} \tag{31}$$

$$h_{r1} = 1.36\cos(\alpha+\beta)\,\alpha^{0.54}E'^{-0.03}D_1^{1.13}(Zl)^{0.13}\{\eta_0\omega\}^{0.7}F^{-0.13}\left(\frac{A_1A_2}{A_1+A_2}\right)^{0.43}\cos^{0.13}\alpha$$

$$h_{r2} = 0.84\cos(\alpha+\beta)\,\alpha^{0.54}E'^{-0.03}D_1^{1.13}(Zl)^{0.13}\{\eta_0\omega\}^{0.7}F^{-0.13}\left(\frac{A_1A_3}{A_3-A_1}\right)^{0.43}\cos^{0.13}(\alpha+2\beta)$$

h_r can be expressed in the form of $h_r = CF_r^{-0.13}$.

where

$$C = \cos(\alpha+\beta)\,\alpha^{0.54}E'^{-0.03}D_1^{1.13}(Zl)^{0.13}\{\eta_0\omega\}^{0.7}\times\left[1.36\left(\frac{A_1A_2}{A_1+A_2}\right)^{0.43}\cos^{0.13}\alpha + 0.84\left(\frac{A_1A_3}{A_3-A_1}\right)^{0.43}\cos^{0.13}(\alpha+2\beta)\right]$$

Therefore, the expression of the approach amount of the roller and the raceway can be obtained:

$$\delta = \delta_r - h_r = nF_r + mF_r\cdot lnF_r - C\cdot F_r^{-0.13} \tag{32}$$

Combining the above Hertz elastic deformation and oil film thickness calculation formula, the comprehensive radial stiffness expression for tapered roller bearing considering oil film thickness can be obtained

$$K = \lim_{\substack{\Delta F_r\to 0\\ \Delta\delta\to 0}}\frac{\Delta F_r}{\Delta\delta} = \lim_{\Delta F_r\to 0}\frac{\Delta F_r}{n\Delta F_r + mF_r\,ln\frac{F_r+\Delta F_r}{F_r} + m\Delta F_r\,ln(F_r+\Delta F_r) + C\left[F_r^{-0.13}-(F_r+\Delta F_r)^{-0.13}\right]} =$$

$$\lim_{\Delta F_r\to 0}\frac{1}{n + mF_r\frac{ln\frac{F_r+\Delta F_r}{F_r}-lnF_r}{\Delta F_r} + m\,ln(F_r+\Delta F_r) + C\left[\frac{F_r^{-0.13}-(F_r+\Delta F_r)^{-0.13}}{\Delta F_r}\right]} \tag{33}$$

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Prabhat Kum