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Dynamic Modelling for Calculating Comprehensive Stiffness of Cylindrical Roller Bearing

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ABSTRACT

The rotor system supported by the rolling bearing is widely used in various fields such as aviation, space, and machinery due to its importance. In the study of the dynamic characteristics for rolling bearings, it is important to accurately calculate the strength of the rolling bearings. The strength of rolling bearings is very important in the analysis of the vibration characteristics of the rotor system. Therefore, in this paper, the method of creating a comprehensive strength model of the rolling bearing is mentioned. Firstly, the contact strength of the rolling bearing was calculated according to the Hertz theory. Here, the radial strength was calculated through the geometric analysis and load analysis of rolling bearings. And then, the thickness of the oil film was determined according to the elastohydrodynamic lubrication theory. Rolling bearings are generally in an elastohydrodynamic lubrication state during the working process, and there is an elastic lubricating film between the rolling elements and the raceway. After determining the thickness of the oil film, the strength of the oil film was determined according to the definition of strength. Then, a comprehensive strength model of the rolling bearing was prepared by synthesizing the deformation stiffness of the bearing and the stiffness of the lubricant film. This dynamic model provides a theoretical basis for the study of the vibration performance of rotor systems supported by rolling bearings.

Keywords : Cylindrical roller bearing, Comprehensive radial stiffness, Structural stiffness, Dynamic characteristic.

I. INTRODUCTION

Bearing-rotor system, as the core component of rotating machinery, plays a very important role in the industrial field. Rotating machinery will have various failures during the work process, causing major economic losses. A large part of it is caused by excessive vibration of the rotor part. The hazards of vibration include noise, damage to the mechanical structure, and even instability of the rotor, fracture of the shaft system, etc., causing major accidents, so the vibration performance of the rotor system has always been a problem. Rotating machinery is generally composed of a bearing and a rotor, and the bearing plays a role in supporting the rotor.

In the study of rotor vibration, the calculation of the stiffness and damping coefficient of the bearing is the key. In 1881, Hertz[1] assumed that the contact area between the rolling elements and the inner and outer rings is an ellipse, and the contact area should be distributed in a semi-ellipsoid. And using the semi-inverse solution method and through the integral transformation, the theoretical solution to the point contact and line contact problems is given, which lays the foundation for the static analysis of rolling bearings. Stribeck[2] derived the relationship between the maximum rolling element load Qmax and the radial load F of the steel ball in 1901. Goodman[3] studied the fatigue failure of the bearing and believed that it was related to the alternating direction of the shear force in the contact area. Palmgren[4] analyzed the bearing deformation and rolling element load distribution under radial force, axial force and moment load, and established the load deformation formula for linear elastic contact problems. Hagiu[5] divides rolling bearing contact into three areas, namely, lubricant inlet area, Hertz contact area and lubricant outlet area. Jones[6] established the ring control theory of high-speed cylindrical roller bearings, which assumes that the steel ball only has pure rolling on a certain raceway. Walters[7] proposed a dynamic analysis model. The analysis model considered the four-degree-of-freedom motion equation of the steel ball and the six-degree-of-freedom motion equation of the cage, using the fourth-order Longge-Kutta method for integration. Venner[8] verified the performance of the elastohydrodynamic lubricant film under time-varying loads under pure rolling conditions. Walford and Stone [9] analyzed in detail the stiffness and damping of the rolling element-raceway contact pair under vibration conditions, and concluded that the stiffness of the contact pair is the Hertz deformation stiffness. Akrurk[10] calculated the influence of the waviness of the inner and outer rings of the cylindrical roller bearing on the vibration and energy of the bearing. McFaddenl[11] studied the influence of single and multiple geometric defects on the raceway on the vibration characteristics of the bearing. H Wu[12] analyzed the contact stress and load distribution between roller elements and raceways using Hertz theory and established the calculation method for bending moment on the bearing end faces.

In this paper, the methodological content for modeling the comprehensive strength of bearing was mentioned based on research and analysis of the preceding literature.

II. Dynamic modelling of the comprehensive stiffness of cylindrical roller bearing

The calculation of the stiffness of the rolling bearing is the basis for analyzing the vibration performance of the supported rotor system. This chapter comprehensively considers Hertz theory and elastohydrodynamic lubrication theory, combined with related stiffness calculation principles, and proposes a comprehensive stiffness model that includes bearing elastic deformation and lubricating oil film lubrication factors under the premise of considering the bearing structure.

2.1 Theoretical basis

˜For cylindrical roller bearings, the contact stiffness will be generated between the roller and the inner and outer rings. If k_1 is the contact stiffness between the roller and the inner ring and k_2 is the contact stiffness between the roller and the outer ring, the radial contact stiffness of the cylindrical roller bearing k_r is as followings:

$$\frac{1}{k_r} = \frac{1}{k_1} + \frac{1}{k_2} \quad (1)$$

˜The existence of the oil film changes the elastic deformation between the roller and the raceway.

When calculating the comprehensive radial stiffness, the radial elastic deformation between the inner and outer rings of the bearing is the radial approach amount after considering the oil film thickness. At this time, the radial load on the bearing is still F_r. According to the definition of stiffness, the comprehensive stiffness of the cylindrical roller bearing can be obtained.

2.2 Dynamic modelling of the comprehensive stiffness

˜Geometric analysis of cylindrical roller bearing

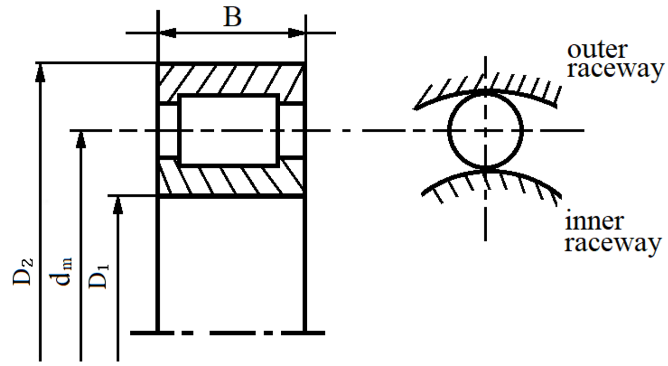

Fig. 1 Geometry diagram of cylindrical roller bearing

Suppose the bearing roller is the contact body I, and the inner ring and the outer ring are the contact body 2. And, the convex surface is defined as a positive value, and the concave surface is defined as a negative value. The main curvature of cylindrical roller bearings can be expressed as:

Roller:

$$\rho_{11} = \rho_{12} = \frac{2}{D_r} \quad (1)$$

where: D_r-roller diameter, mm

Inner ring:

$$\rho_{21} = \frac{2}{D_1}, \rho_{22} = 0 \quad (2)$$

where: D_1- diameter of inner raceway, mm

Outer ring:

$$\rho_{21} = \frac{2}{D_2}, \rho_{22} = 0 \quad (3)$$

where: D_2-diameter of outer raceway, mm

The principal curvature of cylindrical roller bearing is followings:

$$\sum \rho = \rho_{11} + \rho_{12} + \rho_{21} + \rho_{22} \quad (4)$$

The principal curvature of bearing roller and inner ring:

$$\sum \rho_1 = \frac{4}{D} + \frac{2}{D_1}$$

The principal curvature of bearing roller and outer ring:

$$\sum \rho_2 = \frac{4}{D} + \frac{2}{D_2}$$

˜Force analysis of cylindrical roller bearing

The load acting on the bearing is transferred from one ring to the other through the rolling elements. Under the action of radial load, the load of each roller in the bearing is not equal.

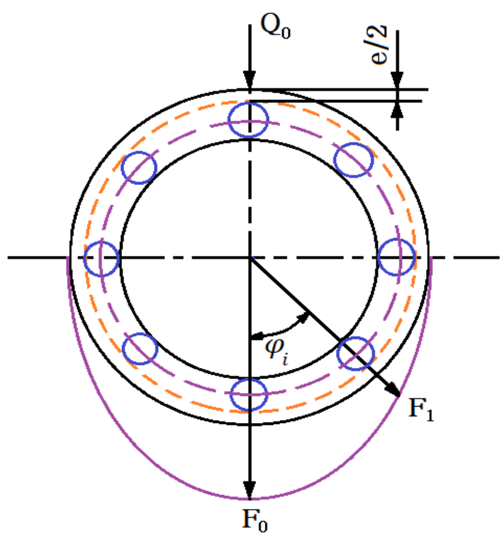

Fig.2 Deformation of cylindrical roller bearing when there is radial displacement

Assume that the radial clearance of the cylindrical roller bearing is e as shown in Fig.2. The Fig. 2 shows that the roller just touches the inner ring of the bearing. When the bearing is under radial load, δ is the total radial displacement of the shaft. On the load line, the maximum elastic deformation of the bearing is δ_{max}, and the deformation at the center angle ψ is as followings:

$$\delta_\psi = \delta cos\psi = \delta_{max} cos\psi \tag{5}$$

where δ_ψ-The angle with the line of action of the radial load is the elastic deformation atψ.

δ_{max}-The amount of elastic deformation where the roller contacts the inner and outer rings along the radial load action line.

ψ- The angle between the center of the roller and the line of action of the radial load.

The relationship between contact load and deformation is as followings:

$$\frac{Q_\psi}{Q_0} = \left(\frac{\delta_\psi}{\delta_{max}}\right)^{1/t} \tag{6}$$

t-The elastic deformation coefficient.

For cylindrical roller bearings 1/t=1.1.

Substituting Eq.(5) into Eq.(6), the contact load at the angle ψ with the line of action of the load is obtained as:

$$Q_\psi = Q_0 \left(\frac{\delta_\psi}{\delta_{max}}\right)^{1/t} = Q_0 (cos\psi)^{1/t} \tag{7}$$

where Q_0-Maximum roller contact load.

Q_ψ-Roller contact load at the angle ψ with the load action line

In Eq. (7), the maximum contact half angle of the bearing can be obtained by setting $Q_\psi = 0$.

$$\psi_l = arccos\psi \tag{8}$$

The balance between the roller contact load Q_ψ and the radial load applied to the bearing is as followings:

$$F_r = Q_0 + 2\sum Q_\psi\, cos\psi$$
(9)

Substitute Eq.(7) into Eq.(9) is as followings.

$$F_r = Q_0 + 2\sum Q_0 (cos\psi)^{1/t}\, cos\psi$$
(10)

The right side of Eq.10 is approximated as an integral form is as followings:

$$F_r = zQ_0J_r$$
(11)

where z-Roller number

J_r- Distribution integral of radial load

$J_r = \left(1 + 2\sum(1 + (cos\psi))^{1+1/t}\right)/z$

According to Eq. 11, the maximum bearing contact load of roller is:

$$Q_0 = \frac{F_r}{zJ_r}$$
(12)

~Calculation of Hertz elastic deformation of cylindrical roller bearing

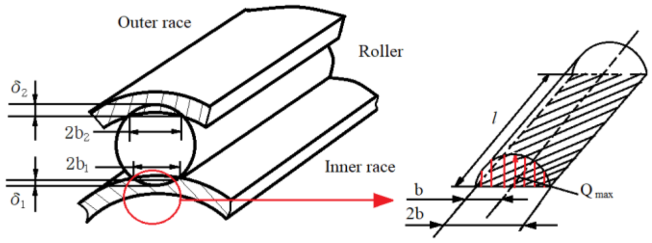

Fig.3 Contact deformation between roller and raceways

As shown in Fig.3, the rollers of a cylindrical roller bearing parallel to each other and the inner and outer ring raceways are in contact with each other due to the load. The contact stress at any point in the contact area is as followings:

$$\sigma = \frac{2Q_0}{\pi bl}\left[1 - \left(\frac{y}{b}\right)^2\right]^{1/2} \tag{13}$$

where b —Contact width, mm

l-Roller length, mm

Q_0-Load on bearing, N

The contact stress is the largest on the center line of the contact area, and its maximum value is as followings:

$$\sigma_{max} = \frac{2Q_0}{\pi bl} \tag{14}$$

The contact width is as followings:

$$\mathrm{b} = \sqrt{\frac{4}{\pi}\left(\frac{1-\nu_1^2}{E_1} - \frac{1-\nu_2^2}{E_2}\right)\frac{q}{\sum\rho}} \tag{15}$$

where q-Load linear density, N/mm

$\sum\rho$-Main curvature sum of the bearing, 1/mm

E_1, E_2-Elastic modulus of materials 1 and 2, Mpa

ν_1, ν_2-Poisson's ratio of materials 1 and 2,

If materials 1 and 2 are the same, the contact width is expressed as:

$$b = 1.59\sqrt{\frac{Q_0}{l}\left(\frac{1-\nu^2}{E}\right)\frac{1}{\sum\rho}} \qquad (16)$$

For internal contact, half width of contact surface is as followings:

$$b_1 = 1.59\sqrt{\frac{Q_0}{l}\left(\frac{1-\nu^2}{E}\right)\frac{1}{\sum\rho}} = 1.59\sqrt{\frac{Q_0}{l}\left(\frac{1-\nu^2}{E}\right)\frac{R_1R_2}{R_2-R_1}} \qquad (17)$$

where R_1-Inner raceway radius, mm

R_2-Outer raceway radius, mm

For external contact, half width of contact surface is as followings:

$$b_2 = 1.59\sqrt{\frac{Q_0}{l}\left(\frac{1-\nu^2}{E}\right)\frac{1}{\sum\rho}} = 1.59\sqrt{\frac{Q_0}{l}\left(\frac{1-\nu^2}{E}\right)\frac{R_1R_2}{R_1+R_2}} \qquad (18)$$

Elastic deformation of roller and inner ring is as followings[13]:

$$\delta_1 = \frac{2Q_0}{\pi l}\left(\frac{1-\nu^2}{E}\right)\left(ln\frac{4R_1R_2}{b_1^2} + 0.814\right) \qquad (19)$$

Elastic deformation of roller and outer ring is as followings[13]:

$$\delta_2 = \frac{Q_0}{l}\left(\frac{1-\nu^2}{E}\right)(1 - lnb_2) \qquad (20)$$

For the radial load F_r, the radial offset of the bearing center in a static state is as followings:

$$\delta = \delta_1 + \delta_2 \qquad (21)$$

For roller and inner raceway, its contact stiffness is as followings:

$$k_1 = \lim_{\substack{\Delta F_r\to 0\\ \Delta\delta_1\to 0}}\frac{\Delta F_r}{\Delta\delta_1} = \left(\frac{d\delta_1}{dF_r}\right)^{-1} = \frac{\pi lEzJ_r}{2(1-\nu^2)\left(ln\frac{4R_1R_2}{b_1^2}+0.814\right)} \qquad (22)$$

For roller and outer raceway, its contact stiffness is as followings:

$$k_2 = \lim_{\substack{\Delta F_r\to 0\\ \Delta\delta_2\to 0}}\frac{\Delta F_r}{\Delta\delta_2} = \left(\frac{d\delta_2}{dF_r}\right)^{-1} = \frac{lEzJ_r}{(1-\nu^2)(1-lnb_2)} \qquad (23)$$

The contact stiffness k_c of the cylindrical roller bearing is equal to the series connection between the stiffness of the inner and outer raceways.

$$\frac{1}{k_c} = \frac{1}{k_1} + \frac{1}{k_2} \qquad (24)$$

˜Oil film thickness calculation of cylindrical roller bearing

The stiffness calculated by Hertz contact deformation is only applicable to static cylindrical roller bearings, only the stiffness generated by the elastic deformation of the rolling elements and raceways is considered, and the stiffness generated by the oil film is not considered. In fact, the rolling bearing is generally in the state of elastic hydrodynamic lubrication during the working process, and there is an elastic lubricating film between the rolling element and the raceway.

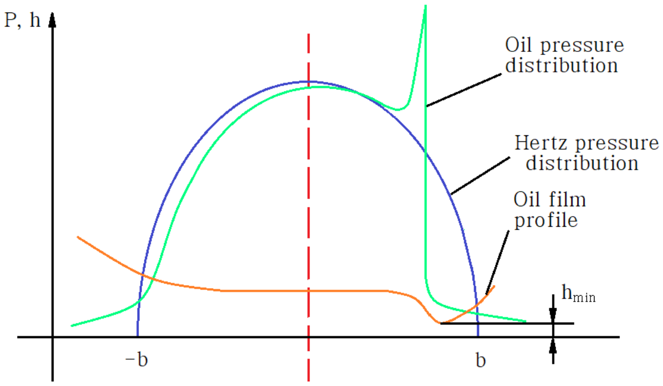

Fig.3 EHL model of cylindrical roller

Minimum oil film thickness of roller and inner raceway is as followings[13]:

$$h_{min1} = 0.28\alpha^{0.54}(\eta_0 n_i)^{0.7} r^{0.43}(R_1 + r)^{0.7}(1-\gamma)^{1.13}(1+\gamma)^{0.7}E'^{-0.03}z^{0.13}l^{0.13}F_r^{-0.13} \qquad (25)$$

Minimum oil film thickness of roller and outer raceway is as followings[13]:

$$h_{min2} = 0.28\alpha^{0.54}(\eta_0 n_i)^{0.7} r^{0.43}(R_1 + r)^{0.7}(1+\gamma)^{1.13}(1-\gamma)^{0.7}E'^{-0.03}z^{0.13}l^{0.13}F_r^{-0.13} \qquad (26)$$

The sum of the minimum oil film thickness formed between roller and inner and outer ring raceway is:

$$h = h_{min1} + h_{min2} = 0.28\alpha^{0.54}(\eta_0 n_i)^{0.7} r^{0.43}(R_1 + r)^{0.7}E'^{-0.03}z^{0.13}l^{0.13}F_r^{-0.13}[(1-\gamma)^{1.13}(1+\gamma)^{0.7} + (1+\gamma)^{1.13}(1-\gamma)^{0.7}] = \chi F_r^{-0.13} \qquad (27)$$

Where $\chi = 0.28\alpha^{0.54}(\eta_0 n_i)^{0.7} r^{0.43}(R_1 + r)^{0.7}E'^{-0.03}z^{0.13}l^{0.13}[(1-\gamma)^{1.13}(1+\gamma)^{0.7} + (1+\gamma)^{1.13}(1-\gamma)^{0.7}]$

In the case of considering elastohydrodynamic lubrication, the calculation expression of the oil film stiffness of the cylindrical roller bearing is:

$$k_{oil} = \lim_{\substack{\Delta F_r\to 0\\ \Delta h\to 0}}\frac{\Delta F_r}{\Delta h} = \left(\frac{dh}{dF_r}\right)^{-1} = (0.13)^{-1}\chi F_r^{-0.13} \qquad (28)$$

˜Comprehensive stiffness calculation of the cylindrical roller bearing

The stiffness of the cylindrical roller bearing is composed of the oil film stiffness and the contact stiffness in series, and the calculation formula for the stiffness considering the elastohydrodynamic lubrication can be deduced as:

$$\frac{1}{k}=\frac{1}{k_c}+\frac{1}{k_{oil}} \tag{29}$$

III. Discussion

Firstly, the principal curvature of the cylindrical roller bearing was calculated, and the dimensions of the contact surface were determined according to Hertz's theory of elastic contact. Next, the load distribution of the cylindrical roller bearing was analyzed and the amount of elastic deformation according to the dimensions of the contact surface was calculated. The contact strength of the rolling bearing was calculated according to the Hertz theory. Here, the radial strength was calculated through the geometric analysis and load analysis of rolling bearings. And then, the thickness of the oil film was determined according to the elastohydrodynamic lubrication theory. Cylindrical roller bearings are generally in an elastohydrodynamic lubrication state during the working process, and there is an elastic lubricating film between the rolling elements and the raceway. After determining the thickness of the oil film, the strength of the oil film was determined according to the definition of strength. Then, a comprehensive strength model of the rolling bearing was prepared by synthesizing the deformation stiffness of the bearing and the stiffness of the lubricant film.

IV. Conclusion

Cylindrical roller bearings, as an important basic component have been widely used in various rotating machinery, and their operating conditions often directly affect the performance of the entire mechanical system. The strength of rolling bearings is very important in the analysis of the vibration characteristics of the rotor system. Therefore, in this paper, the method of creating a comprehensive strength model of the rolling bearing is mentioned. Firstly, the contact strength of the rolling bearing was calculated according to the Hertz theory. Here, the radial strength was calculated through the geometric analysis and load analysis of rolling bearings. And then, the thickness of the oil film was determined according to the elastohydrodynamic lubrication theory. Rolling bearings are generally in an elastohydrodynamic lubrication state during the working process, and there is an elastic lubricating film between the rolling elements and the raceway. After determining the thickness of the oil film, the strength of the oil film was determined according to the definition of strength. Then, a comprehensive strength model of the rolling bearing was prepared by synthesizing the deformation stiffness of the bearing and the stiffness of the lubricant film. This dynamic model provides a theoretical basis for the study of the vibration performance of rotor systems supported by rolling bearings.

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