

## Dynamic Modeling of the Blade-Rotor-Shaft-Bearing System with SFD

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### ABSTRACT

There are often a large number of blades on the rotating shafts of rotating machinery such as steam turbine engines, aero engines, and compressors. In actual work, forces are transferred between the shafting and the blades. However, most of the current researches separate the blade from the rotor-bearing system, and study the vibration characteristics of the blade system and the nonlinear behavior of the rotor system separately. As a result, the interaction between the blade and the rotor-bearing system is fragmented, and the coupling vibration between the blade-disk-bearing system is ignored, which will inevitably affect the accuracy of the analysis. With the gradual increase of modern rotating machinery's high speed, light structure and the gradual increase of blade length, the degree of coupling between the blade and the rotor-bearing system has become tighter. Therefore, it is necessary to study the coupling effect of blade, disc and rotor vibration. At the same time, there are a lot of nonlinear phenomena in the rotor during operation. However, there are still relatively few studies on the nonlinearity of the blade-rotor-bearing system. A research project was carried out to create a dynamic equation of a blade-rotor-bearing-shaft system. Firstly, the model of a BRSB system is developed with consideration the rubbing between the blade-tip and casing by using contact dynamics theory. Next, Timoshenko beam elements are adopted to simulate the shaft and the blade, and shell elements to simulate the disk, and spring-damping elements to simulate the ball bearings. This dynamic modelling has certain significance in analysing the nonlinear characteristics and improving the stability of the rotor system.

**Keywords** : Dynamic Modelling, Squeeze Film Damper, Blade-Rotor-Shaft-Bearing System, Nonlinear Characteristics, Stability.

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## I. INTRODUCTION

As a kind of widely used mechanical equipment, high-speed rotating machinery plays an important

role in many industries such as aerospace, electric power, metallurgy, etc. The rotor system is an important part of high-speed rotating machinery, and its research has always received wide attention. In

recent years, the vicious accidents caused by the failure of the rotor system have caused great losses to the enterprise. Therefore, accurately modeling a rotor system and accurately analyzing its dynamic characteristics has attracted increasing attention. However, most of the current researches separate the blade from the rotor-bearing system, and study the vibration characteristics of the blade system and the nonlinear behavior of the rotor system separately. The predecessors first conducted a study on the creation of dynamic models and nonlinear characteristics analysis of the rotor-bearing system. Nelson[1] obtained the finite element memory and mass matrix of the Rayleigh beam-shaft model by calculating the gyroscopic effect and rotational inertia of the rotating shaft. Based on this, the finite element formula of the Timoshenko beam-shaft model was derived [2]. Oncescu[3] studied and analyzed the stability of an asymmetric rotor-bearing system by combining the finite element method and the time transfer function method based on the Floquet theory. Mohiuddin[4] studied the stability problem of the rotor-bearing system by using the mode reduction method by applying the finite element method and the Lagrange method at the same time. Friswell[5] used the finite element method to model the rotor system and calculated the critical speed of the rotor bearing system in consideration of the bearing dynamic characteristic coefficient according to the change of rotational speed. D.Mku[6] used Timoshenko beam elements to determine the critical speed and stability of the rotor-bearing system in various states. Madhumita[7] calculated the critical speed of the rotor system through the cambell diagram and studied the stability of the damped rotor bearing system based on the finite element method. A mathematical model of a lightweight flexible rotor disk bearing system with geometric eccentricity and mass imbalance was created. This mathematical model includes a bidirectional flexible shaft

characterized by nonlinear curvature and gyroscopic effect, geometric eccentricity, a rigid disk crooked with unbalance mass, and nonlinear flexible bearings [8]. Next, a literature study was conducted on the creation of a dynamic model and analysis of the vibration characteristics of the blades of rotor system. To study the nonlinear dynamic behavior of the bladed overhang rotor system with squeeze film damper (SFD), a blade-overhang rotor-SFD model is formulated using the lumped mass method and the Lagrange approach[9]. The continuum model of flexible blade-rotor-bearing coupling system is established, simplifying the shaft as Timoshenko beam and the Lagrange method is utilized to derive the differential equation of motion of system[10]. The finite element (FE) model of an SDB system is developed by using contact dynamics theory[11]. Therefore, in this work, based on the detailed research and analysis of the preceding literature, the method of establishing the dynamic equation of the BRSB system was described.

## II. Dynamic equation of the blade-rotor-bearing coupling system

### 2.1 Basic assumptions for building the model

The following assumptions are made to model the blade-rotor-bearing coupling system.

- ① The material is an isotropic linear elastic material.
- ② Ignore the contact relationship between the blade and the wheel, and the wheel and the shaft, and assume that the three are rigidly connected.
- ③ The roulette is simplified to a rigid disk, and the elastic deformation of the roulette is not considered.
- ④ The bearing is simulated with linear stiffness and damping.

- ⑤ The blades installed on the roulette are exactly the same.
- ⑥ All blades have the same parameters.
- ⑦ Only the bending deformation of the blade is considered, and the radial displacement and torsional displacement of the blade are ignored.
- ⑧ Assuming that the bending deformation direction of the blade is the same or opposite to the shaft twisting direction, and the rotation angle is small, the out-of-plane rotation angle of the blade is ignored.

## 2.2 Dynamic model of blade-rotor-bearing coupling system

Fig. 1 is a schematic diagram of the blade- rotor-bearing coupling system model. The bending-torsional coupling vibration of the rotating shaft and the radial and transverse vibration of the blade are considered. The rotor system consists of a rotating shaft, blades, bearings and a rigid disk. The blade is simulated using a cantilever Timoshenko flexible beam model.

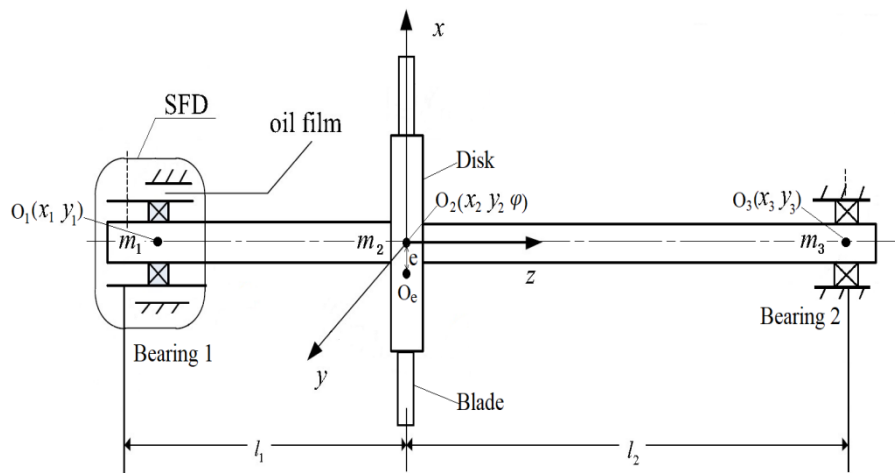


Fig 1. Model of the blade-rotor-bearing system

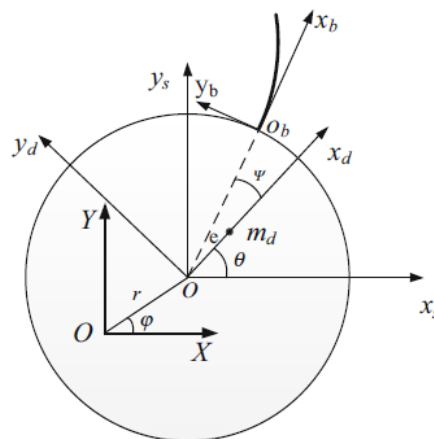


Fig 2. Model of the blade system

Where  $e$  is the eccentricity,

$x_d, y_d$  are the displacements of the disk's mass center in the horizontal and vertical directions, respectively;  
 $x_s, y_s$  are the displacements of disk's centroid in the horizontal and vertical directions, respectively,

0 is the rotational angle of disk's mass center.

### 2.3 Differential equation of motion of blade-rotor-bearing coupling system

#### 2.3.1 Kinetic energy of blade-rotor-bearing coupling system ~ Kinetic energy of disk

$$T_{disk} = \frac{1}{2}m_2(\dot{x}_c^2 + \dot{y}_c^2) \tag{1}$$

Where  $(x_c, y_c)$  is the center of mass coordinates of the disk.

$e$  is the eccentricity.

$\psi$  is the angle of rotation of the rotor.

$m_2$  is the lumped mass at the disk.

$$x_c = x_2 + e\cos\psi, y_c = y_2 + e\sin\psi$$

$$T_{disk} = \frac{1}{2}m_2(\dot{x}_c^2 + \dot{y}_c^2) = \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2 + e^2\dot{\psi}^2 + 2e\dot{\psi}\dot{y}_2\cos\psi - 2e\dot{\psi}\dot{x}_2\sin\psi) \tag{2}$$

#### ~ Kinetic energy of bearings

$$T_{bearings} = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_3(\dot{x}_3^2 + \dot{y}_3^2) \tag{3}$$

Where  $m_1, m_3$  is the lumped mass at the left and right bearing.

#### ~ Rotational kinetic energy of the rotor

$$T_{rotation} = \frac{1}{2}J_c\dot{\psi}^2 \tag{4}$$

Where  $J_c$  is the rotational inertia of the rotor.

When the system rotates steadily,  $\psi = \Omega t + \varphi$

$\Omega$  is the angular velocity of the rotation of the disk.

$\varphi$  is the torsion angle of the disk.

Due to the existence of mass eccentricity, from the parallel axis theorem:

$$J_c = J + m_2e^2$$

Thus, the expression of rotational kinetic energy can be obtained as:

$$T_{rotation} = \frac{1}{2}J_c\dot{\psi}^2 = \frac{1}{2}(J + m_2e^2)\dot{\psi}^2 \tag{5}$$

#### ~ Kinetic energy of blades

According to the rotor dynamics theory, the kinetic energy of the blade is as followings:

$$T_{blades} = \sum_{i=1}^n \frac{1}{2} \int_0^{l_b} \rho_b A_b \dot{R}_{qi}^T \dot{R}_{qi} dx \tag{6}$$

Where  $\rho_b$  is the blade density.

$A_b$  is the blade cross-sectional area.

$$A_b = A_0(1 - \tau_b \frac{x}{L})(1 - \tau_h \frac{x}{L})$$

$A_0$  is the cross-sectional area at the blade root.

$$A_0 = b_0 h_0$$

$\tau_b$  and  $\tau_h$  are the taper ratios in the width direction and the thickness direction, respectively.

$$\tau_b = 1 - \frac{b_1}{b_0}, \tau_h = 1 - \frac{h_1}{h_0}$$

$b_1$  and  $h_1$  are the blade width and thickness at the blade tip, respectively.

$b_0$  and  $h_0$  are the blade width and thickness at the blade root, respectively.

$R_{qi}$  is the displacement of any point  $q$  on the  $i$ -th blade under the fixed coordinate system after bending and deformation, which is expressed as:

$$R_{qi} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + [A_1(\varphi)][A_2(\eta)][A_3(\vartheta_i)]r_q^d \tag{7}$$

Where  $[A_1(\varphi)]$ ,  $[A_2(\eta)]$  are the conversion matrices from the disk coordinate system  $x^d, y^d$  to the inertial coordinate system

$\eta$  is the rotation angle of disk.

$[A_3(\vartheta_i)]$  is the conversion matrix of the  $i$ -th blade relative to the blade whose angle is zero.

$\vartheta_i = 2\pi(i - 1)/n$  describes the position of the  $i$ -th blade on the disk.

$r_q^d$  is the coordinates of point  $q$  in the disk coordinate system  $x^d, y^d$ .

$$r_q^d = (x + R_d)i + u(x, t)j$$

Therefore

$$R_{qi} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + [A_1(\varphi)][A_2(\eta)][A_3(\vartheta_i)] \begin{bmatrix} (x + R_d) \\ u(x, t) \end{bmatrix} \tag{8}$$

Where  $R_d$  is the radius of the disk.

$x$  and  $u(x, t)$  are the axial position of the blade and the displacement of the blade in the blade coordinate system  $x^d$  and  $y^d$ , respectively.

The transformation matrix  $[A_1(\varphi)]$ ,  $[A_2(\eta)]$  and  $[A_3(\vartheta_i)]$  are respectively:

$$[A_1(\varphi)] = \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix}, [A_2(\eta)] = \begin{bmatrix} \cos \eta & -\sin \eta \\ \sin \eta & \cos \eta \end{bmatrix}, [A_3(\vartheta_i)] = \begin{bmatrix} \cos \vartheta_i & -\sin \vartheta_i \\ \sin \vartheta_i & \cos \vartheta_i \end{bmatrix}$$

Considering that the torsion angle  $\varphi$  is very small and  $\eta = \Omega t$ , we can write as following:

$$[A_1(\varphi)] = \begin{bmatrix} 1 & -\varphi \\ \varphi & 1 \end{bmatrix}, [A_2(\eta)] = \begin{bmatrix} \cos \Omega t & -\sin \Omega t \\ \sin \Omega t & \cos \Omega t \end{bmatrix}$$

Substituting the transformation matrix, the velocity expression of the  $i$ -th blade can be obtained as:

$$\dot{R}_{qi} = \begin{bmatrix} \dot{x}_2 - \alpha \cos \theta_i - \beta \sin \theta_i \\ \dot{y}_2 + \beta \cos \theta_i - \alpha \sin \theta_i \end{bmatrix} \tag{9}$$

Where  $\theta_i = \Omega t + \vartheta_i$

$$\alpha = \dot{\theta}[\varphi(x + R_d) + u] + \dot{u}\varphi + \dot{\varphi}u$$

$$\beta = \dot{\theta}[(x + R_d) - \varphi u] + \dot{u} + \dot{\varphi}(x + R_d)$$

Therefore

$$\begin{aligned} \dot{R}_{qi}^T \dot{R}_{qi} &= (\dot{x}_2 - \alpha \cos \theta_i - \beta \sin \theta_i)^2 + (\dot{y}_2 + \beta \cos \theta_i - \alpha \sin \theta_i)^2 = \dot{x}_2^2 + \alpha^2 \cos^2 \theta_i + \beta^2 \sin^2 \theta_i - \\ &- 2\dot{x}_2 \alpha \cos \theta_i - 2\dot{x}_2 \beta \sin \theta_i + 2\alpha \beta \sin \theta_i \cos \theta_i + \dot{y}_2^2 + \beta^2 \cos^2 \theta_i + \alpha^2 \sin^2 \theta_i + 2\dot{y}_2 \beta \cos \theta_i - \\ &2\dot{y}_2 \alpha \sin \theta_i - 2\alpha \beta \sin \theta_i \cos \theta_i \end{aligned} \tag{10}$$

Since the blades are evenly and symmetrically distributed on the disk, there is the following triangular equation relationship

$$\sum_{i=1}^n \sin \theta_i = \sum_{i=1}^n \cos \theta_i = 0$$

$$\sum_{i=1}^n \sin \theta_i \cos \theta_i = 0$$

$$\sum_{i=1}^n \sin^2 \theta_i = \sum_{i=1}^n \cos^2 \theta_i = \frac{n}{2} \tag{11}$$

Considering the case of four blades, and substituting Eq. (10) into Eq. (6), the kinetic energy of the blades can be expressed as:

$$T_{blades} = \sum_{i=1}^n \frac{1}{2} \int_0^{l_b} \rho_b A_b \dot{R}_{qi}^T \dot{R}_{qi} dx = \frac{n}{2} \Omega^2 \varphi^2 \int_0^{l_b} \rho_b A_b (x + R_d)^2 dx + \frac{n}{2} (\Omega + \dot{\varphi})^2 \int_0^{l_b} \rho_b A_b (x + R_d)^2 dx + \frac{n}{2} [(\Omega + \dot{\varphi})^2 + \varphi^2 \Omega^2] \int_0^{l_b} \rho_b A_b u^2 dx + \frac{n}{2} (1 + \varphi^2) \int_0^{l_b} \rho_b A_b \dot{u}^2 dx + n\varphi\dot{\varphi} \int_0^{l_b} \rho_b A_b u \dot{u} dx + n[\Omega(1 + \varphi^2) + \dot{\varphi}] \int_0^{l_b} \rho_b A_b (x + R_d) \dot{u} dx + \frac{n}{2} \rho_b A_b l_b (\dot{x}_2^2 + \dot{y}_2^2) \quad (12)$$

### 2.3.2 Potential energy of blade-rotor-bearing coupling system

#### ~ Potential energy of disk

$$V_{disk} = \frac{1}{2} k_2 (x_2^2 + y_2^2) + m_2 g (y_2 + e \sin \psi) + \frac{1}{2} k_\varphi \varphi^2 \quad (13)$$

Where  $k_2$  is the bending stiffness.

$k_\varphi$  is the torsional stiffness.

#### ~ Potential energy of bearings

$$V_{bearings} = \frac{1}{2} k_1 (x_1^2 + y_1^2) + m_1 g y_1 + \frac{1}{2} k_3 (x_3^2 + y_3^2) + m_3 g y_3 \quad (14)$$

Where  $k_1$  is the bending stiffness.

#### ~ Potential energy of blades

① The strain energy of a blade due to bending

$$V_{Bending} = \sum_{i=1}^n \int_0^{l_b} EI \left( \frac{\partial^2 u}{\partial x^2} \right)^2 dx = n \int_0^{l_b} EI \left( \frac{\partial^2 u}{\partial x^2} \right)^2 dx \quad (15)$$

② Potential energy of axial contraction due to transverse deformation

The axial shrinkage due to the bending deformation of the blade can be estimated as:

$$d_\delta \cong -\frac{1}{2} \left( \frac{\partial u}{\partial x} \right)^2 dx \quad (16)$$

The inertial force generated at point q due to the rotational movement of the blade can be expressed as:

$$F_q = \int_x^L \rho_b (x + R_d) (\Omega + \dot{\varphi})^2 dx \quad (17)$$

The axial contraction potential energy can be expressed as:

$$V_{Aj} = \int_0^{l_b} F_q d\delta \quad (18)$$

Substituting (16) and (17) into (18) can be obtained as follows:

$$V_A = \sum_{i=1}^n \frac{1}{2} (\Omega + \dot{\varphi})^2 \left\{ \frac{1}{2} \int_0^{l_b} \rho_b A_b (l^2 - x^2) \left( \frac{\partial u}{\partial x} \right)^2 dx + R_d \int_0^{l_b} \rho_b A_b (l - x) \left( \frac{\partial u}{\partial x} \right)^2 dx \right\} = \frac{n}{2} (\Omega + \dot{\varphi})^2 \left\{ \frac{1}{2} \int_0^{l_b} \rho_b A_b (l^2 - x^2) \left( \frac{\partial u}{\partial x} \right)^2 dx + R_d \int_0^{l_b} \rho_b A_b (l - x) \left( \frac{\partial u}{\partial x} \right)^2 dx \right\} \quad (19)$$

### 2.3.3 Kinetic energy and potential energy of the system

#### ~ Kinetic energy of the system

Summing the kinetic energy of the rotor with the kinetic energy of all blades, the kinetic energy of the system can be obtained as:

$$T_{kinetic} = T_{rotor} + T_{Blade} = T_{disk} + T_{bearings} + T_{rotation} + T_{blades} = \frac{1}{2} m_2 (\dot{x}_c^2 + \dot{y}_c^2) = \frac{1}{2} m_2 (\dot{x}_c^2 + \dot{y}_c^2 + e^2 \dot{\psi}^2 + 2e\dot{\psi}\dot{y}_c \cos \psi - 2e\dot{\psi}\dot{x}_c \sin \psi) + \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m_3 (\dot{x}_3^2 + \dot{y}_3^2) + \frac{1}{2} (J + m_2 e^2) \dot{\psi}^2 + \frac{n}{2} \Omega^2 \varphi^2 \int_0^{l_b} \rho_b A_b (x + R_d)^2 dx + \frac{n}{2} (\Omega + \dot{\varphi})^2 \int_0^{l_b} \rho_b A_b (x + R_d)^2 dx + \frac{n}{2} [(\Omega + \dot{\varphi})^2 + \varphi^2 \Omega^2] \int_0^{l_b} \rho_b A_b u^2 dx + \frac{n}{2} (1 + \varphi^2) \int_0^{l_b} \rho_b A_b \dot{u}^2 dx + n\varphi\dot{\varphi} \int_0^{l_b} \rho_b A_b u \dot{u} dx + n[\Omega(1 + \varphi^2) + \dot{\varphi}] \int_0^{l_b} \rho_b A_b (x + R_d) \dot{u} dx + \frac{n}{2} \rho_b A_b l_b (\dot{x}_2^2 + \dot{y}_2^2) \quad (20)$$

~ **Potential energy of the system**

Summing the potential energy of the rotor with the potential energy of all blades, the potential energy of the system can be obtained as:

$$V_{potential} = V_{rotor} + V_{Blade} = V_{disk} + V_{bearings} + (V_{Bending} + V_A) = \frac{1}{2}k_2(x_2^2 + y_2^2) + m_2g(y_2 + esin\psi) + \frac{1}{2}k_\phi\phi^2 + \frac{1}{2}k_1(x_1^2 + y_1^2) + m_1gy_1 + \frac{1}{2}k_3(x_3^2 + y_3^2) + m_3gy_3 + n \int_0^{l_b} EI \left(\frac{\partial^2 u}{\partial x^2}\right)^2 dx + \frac{n}{2}(\Omega + \dot{\phi})^2 \left\{ \frac{1}{2} \int_0^{l_b} \rho_b A_b (l^2 - x^2) \left(\frac{\partial u}{\partial x}\right)^2 dx + R_d \int_0^{l_b} \rho_b A_b (l - x) \left(\frac{\partial u}{\partial x}\right)^2 dx \right\} \tag{21}$$

**2.3.4 Generalized forces and generalized displacement of blade-rotor-bearing coupling system**

① **Generalized forces of blade-rotor-bearing coupling system**

~ **Generalized force of the rotor**

Considering the influence of the damping system, the generalized force can be expressed as:

$$\begin{aligned} Q_{x_1} &= -c_1 \dot{x}_1 \\ Q_{y_1} &= -c_1 \dot{y}_1 \\ Q_{x_2} &= -c_2 \dot{x}_2 \\ Q_{y_2} &= -c_2 \dot{y}_2 \\ Q_\phi &= -c_t \dot{\phi} \\ Q_{x_3} &= -c_3 \dot{x}_3 \\ Q_{y_3} &= -c_3 \dot{y}_3 \end{aligned} \tag{22}$$

~ **Nonlinear oil film force**

$$\begin{cases} F_x = F_r \frac{x+l_1\phi_y+x_0}{r} - F_t \frac{y-l_1\phi_x-y_0}{r} \\ F_y = F_r \frac{y-l_1\phi_x-y_0}{r} + F_t \frac{x+l_1\phi_y+x_0}{r} \end{cases} \tag{23}$$

Where  $F_r$  is the radial force of oil film.

$F_t$  is the tangential radial force of oil film.

~ **Generalized forces of blade-rotor-bearing coupling system**

$$F = \{F_x, F_y, -G_1, F_{xc}, F_{yc}, -G_2, -G_3\} \tag{24}$$

Where  $F_x$  and  $F_y$  are the non-linear oil film forces in the x and y directions at the bearing, respectively.

$F_{xc}$  and  $F_{yc}$  are the centrifugal force in the x and y directions of the disk, and  $G_i$  ( $i=1,2,3$ ) are the gravity at each node.

② **Generalized displacement of blade-rotor-bearing coupling system**

$$D = \{x_1, y_1, x_2, y_2, \phi, x_3, y_3\} \tag{25}$$

**2.3.5 Differential equation of motion of blade-rotor-bearing coupling system**

The Lagrange equation is used to establish the dynamic equation of the system.

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{D}} \right) - \frac{\partial L}{\partial D} = - \frac{\partial H}{\partial D} + F \tag{26}$$

Where  $L=T-V$  is Lagrange function of the system,

$T$  and  $V$  are the kinetic energy and potential energy of the system respectively,

$H$  is the dissipation function of the system,

$F$  is the generalized force of the system, and  $D$  is the generalized coordinates of the system.

Substitute the kinetic energy, potential energy and generalized force of the system into Lagrange equation to obtain the differential equation of motion of the system under generalized coordinates.

$$\begin{aligned}
 m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1(x_1 - x_2) &= F_x \\
 m_1\ddot{y}_1 + c_1\dot{y}_1 + k_1(y_1 - y_2) &= F_y - m_1g \\
 m_3\ddot{x}_3 + c_3\dot{x}_3 + k_3(x_3 - x_2) &= F_x \\
 m_3\ddot{y}_3 + c_3\dot{y}_3 + k_3(y_3 - y_2) &= F_y - m_3g \\
 (m_2 + m_b)\ddot{x}_2 + c_2\dot{x}_2 + k_2(x_2 - x_1) &= (m_2 + m_b)e[(\Omega + \dot{\varphi})^2 \cos(\Omega t + \varphi) + \ddot{\varphi} \sin(\Omega t + \varphi)] \\
 (m_2 + 4\rho_b A_b l_b)\ddot{y}_2 + c_2\dot{y}_2 + k_2(y_2 - y_1) &= (m_2 + m_b)e[(\Omega + \dot{\varphi})^2 \sin(\Omega t + \varphi) - \ddot{\varphi} \cos(\Omega t + \varphi)] - (m_2 + m_b)g \\
 [J_t + \frac{nm_b}{2}(r_d + l_b)^2]\ddot{\varphi}_x + [J_p + nm_b(r_d + l_b)^2]\Omega\dot{\varphi}_y + m_b(r_d + l_b)l_b \sin\Omega t \sum_{i=1}^n (\Omega^2\phi_i + \ddot{\phi}_i) \cos\vartheta_i \\
 + m_b(r_d + l_b)l_b \cos\Omega t \sum_{i=1}^n (\Omega^2\phi_i + \ddot{\phi}_i) \sin\vartheta_i + c_2\dot{y}_2 + c_2\dot{\varphi}_x + k_2y_2 + k_3\varphi_x &= 0 \\
 [J_t + \frac{nm_b}{2}(r_d + l_b)^2]\ddot{\varphi}_y - [J_p + nm_b(r_d + l_b)^2]\Omega\dot{\varphi}_x - m_b(r_d + l_b)l_b \cos\Omega t \sum_{i=1}^n (\Omega^2\phi_i + \ddot{\phi}_i) \cos\vartheta_i \\
 + m_b(r_d + l_b)l_b \sin\Omega t \sum_{i=1}^n (\Omega^2\phi_i + \ddot{\phi}_i) \sin\vartheta_i + c_2\dot{x}_2 + c_2\dot{\varphi}_y + k_2y_2 + k_3\varphi_y &= 0
 \end{aligned}
 \tag{27}$$

### III. CONCLUSION

With the gradual increase of modern rotating machinery's high speed, light structure and the gradual increase of blade length, the degree of coupling between the blade and the rotor-bearing system has become tighter. Therefore, it is necessary to study the coupling effect of blade, disc and rotor vibration. At the same time, there are a lot of nonlinear phenomena in the rotor during operation. However, there are still relatively few studies on the nonlinearity of the blade-rotor-bearing system. A research project was carried out to create a dynamic equation of a blade-rotor-bearing-shaft system. Firstly, the model of a BRSB system is developed with consideration the rubbing between the blade-tip and casing by using contact dynamics theory. Next, Timoshenko beam elements are adopted to simulate the shaft and the blade, and shell elements to simulate the disk, and spring-damping elements to simulate the ball bearings. This dynamic modelling has certain

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### IV. REFERENCES

- [1]. Nelson H D, McVaugh J M. The dynamics of rotor-bearing systems using finite elements]. 1976.
- [2]. Nelson H D. A finite rotating shaft element using Timoshenko beam theory]. 1980.
- [3]. Oncescu F, Lakis A A, Ostiguy G. Investigation of the stability and steady state response of asymmetric rotors, using finite element formulation]. Journal of Sound and Vibration, 2001, 245(2): 303-328.
- [4]. Mohiuddin M A, Bettayeb M, Khulief Y A. Dynamic analysis and reduced order modelling of flexible rotor-bearing systems]. Computers & structures, 1998, 69(3): 349-359.
- [5]. Friswell M I, Garvey S D, Penny J E T, et al. Computing critical speeds for rotating machines



- with speed dependent bearing properties]].  
Journal of sound and vibration, 1998, 213(1):  
139-158.
- [6]. Ku D M. Finite element analysis of whirl speeds for rotor-bearing systems with internal damping]]. Mechanical Systems and Signal Processing, 1998, 12(5): 599-610.
- [7]. Kalita M, Kakoty S K. Analysis of whirl speeds for rotor-bearing systems supported on fluid film bearings]]. Mechanical Systems and Signal Processing, 2004, 18(6): 1369-1380.
- [8]. Phadatare H P, Pratiher B. Nonlinear modeling, dynamics, and chaos in a large deflection model of a rotor-disk-bearing system under geometric eccentricity and mass unbalance]]. Acta Mechanica, 2020, 231(3): 907-928.
- [9]. Cao D Q, Wang L G, Chen Y S, et al. Bifurcation and chaos of the bladed overhang rotor system with squeeze film dampers]]. Science in China Series E: Technological Sciences, 2010(03):709-720.
- [10]. Li C, She H, Tang Q, et al. The effect of blade vibration on the nonlinear characteristics of rotor-bearing system supported by nonlinear suspension]]. Nonlinear Dynamics, 2017.
- [11]. B H M , A Y L , A Z W , et al. Vibration response analysis of a rotational shaft-disk-blade system with blade-tip rubbing]]. International Journal of Mechanical Sciences, 2016, 107:110-125.

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