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Dynamic Modelling of the Rotor System with the Squeeze Film Damper

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ABSTRACT

The main function of extruded oil film damper is to reduce the amplitude of the supporting external force and the resonance of the rotor. Because of its simple structure, light weight, small volume and good vibration reduction performance, it has been widely used in modern aero-engines. As the basic structure of the engine, the rotor system is the main source of engine vibration due to its unbalance characteristics and vibration in the critical speed region. Therefore, it has become a trend to equip the extruder film damper on the aero-engine. Generally, due to the influence of processing technology, structure dead weight, structure design and assembly technology, the extruded oil film damper itself has static eccentricity, which affects the stable operation of the system to a certain extent. Therefore, the theoretical research on the vibration characteristics of the extruded oil film damper rotor system has very important engineering value and practical significance. Therefore, in this dissertation, the task of research was to create a mathematical model of the rotor system with and without the squeeze film damper. First, the model of a single rotor system with and without the squeeze film damper was made. The modeling method of a simple single rotor system with one disk and two supports was specifically mentioned. Next, the mathematical modelling method of the dual rotor system with and without the squeeze film damper was mentioned. The dual rotor system is composed of a low-pressure rotor part and a high-pressure rotor part. Firstly, the mathematical model of low pressure rotor system of double rotor system was built. In modelling, the effects of asymmetry, mass eccentricity, and gyroscopic effect of the low-pressure rotor system and the squeeze film damper were fully considered and the differential equation of the low-pressure rotor system was written by utilizing Lagrange equation. Next, a differential equation of a high pressure rotor system was formulated, considering sufficient conditions such as low pressure rotor system.

Keywords : Squeeze film damper, Dual rotor system, Single rotor system, Dynamic modelling, Rolling bearing.

I. INTRODUCTION

As the core rotating component of power machinery and working machinery, the rotor system is widely used in the fields of transportation, machinery, electric power, chemical industry, and aviation. When it rotates at certain speeds, it will undergo relatively large deformation and cause system resonance, causing resonance. The speed at that time is called the critical speed of the rotor system. Compared with the single-rotor system, the dual-rotor system can work under better conditions. Compared with the three-rotor system, the number of rotors is small, the structure is simple, and the operation is convenient, so that the turbojet It is widely used in turbofan engine systems. The unbalanced characteristics of the rotor system itself and the vibration in the critical speed zone are the main sources of engine vibration, and its dynamic characteristics have a greater impact on the performance of aeroengines. Therefore, in order to improve the performance of aeroengines, the overall maneuverability and the study of the dynamic characteristics of the rotor system are an effective way to solve the engine vibration problem. In essence, most of the problems of the rotor system, such as the friction between the moving and static parts of the rotor, the crack of the rotor, the misalignment, the loosening of the support, etc., belong to the nonlinear theory, especially with the development of science and technology and production, the rotor system is non-linear. The study of linear dynamics has also attracted more and more attention, and the use of nonlinear theory to study the nonlinear vibration of the rotor system has become a very important task. Dutt[1] used the finite element method to create a model of the rotor bearing support system under the condition that the mass of the support was considered and the mass of the rotor was ignored. Kang[2] created a three-dimensional finite element model of the rotor, bearing, and support, respectively, and studied the relationship between the support type and the stability of the rotor system. Nelson[3] obtained the finite element memory and mass matrix of the Rayleigh beam-shaft model by calculating the gyroscopic effect and rotational inertia of the rotating shaft. Based on this, the finite element formula of the Timoshenko beam-shaft model was derived [4]. Oncescu[5] studied and analyzed the stability of an asymmetric rotor-bearing system by combining the finite element method and the time transfer function method based on the Floquet theory. Mohiuddin[6] studied the stability problem of the rotor-bearing system by using the mode reduction method by applying the finite element method and the Lagrange method at the same time. Friswell[7] used the finite element method to model the rotor system and calculated the critical speed of the rotor bearing system in consideration of the bearing dynamic characteristic coefficient according to the change of rotational speed. D.Mku[8] used Timoshenko beam elements to determine the critical speed and stability of the rotor-bearing system in various states. Madhumita[9] calculated the critical speed of the rotor system through the cambell diagram and studied the stability of the damped rotor bearing system based on the finite element method. Pingchao Yu[10] studied the dynamic responses of the complicated dual-rotor systems at instantaneous and windmilling statuses when FBO event occurs. A simplified dynamic model of a dual-rotor system coupled with blade disk is built by Zhenyong Lu[11]. Harris[12] specifically mentioned the nonlinear dynamic modeling method in SFD. Linnett[13] studied that when the ratio of the oil film gap is below 0.4, the nonlinearity of the system is weak, and it can usually be linearized, and the damper can work under better conditions. Fleming[14] studied that when the unbalanced amount of the system is greater than the design value, the static eccentricity ratio of the SFD is greater than 0.4, and the jumping phenomenon may occur in the rotor system at this time. Sykes[15] found that the SFD static eccentric condition will cause the rotor system to produce sub-harmonic vibration. In view of the above characteristics of SFD, it is necessary to deeply analyze its vibration reduction mechanism, reasonably design the parameter range of the response, and improve the working characteristics of the damper so that it can maintain a better working state[16]. Chang[17] concluded that PSFD can effectively suppress the dynamic amplitude of the system and improve the dynamic stability of the gear bearing system. Chen[18] studied the influence of the dual-frequency excitation ratio and the squeeze film damper parameter changes on the flexural mode bifurcation characteristics of the system, and verified the sensitivity of the system to the dual-frequency excitation ratio. Wang[19] studied designed and experimental investigations on dynamics of squeeze film damper (SFD) under the different additional centrifugal force and gyroscopic moment are carried out. Zhou[20] established a dynamic model of a double-gap squeeze film damper rotor system, and verified the authenticity of the simulation results with experiments. In this paper, a mathematical model of the rotor system with a squeeze film damper was created based on the research and analysis of the precedence literatures in detail.

II. DYNAMIC MODELLING OF SINGLE ROTOR SYSTEM

2.1 Dynamic modelling of single rotor system without squeeze film damper (SFD)

2.1.1 Model of single rotor system without squeeze film damper (SFD)

The rotor system consists of a turntable and two supports, and a single disc with eccentric mass is placed in the center of the shaft.

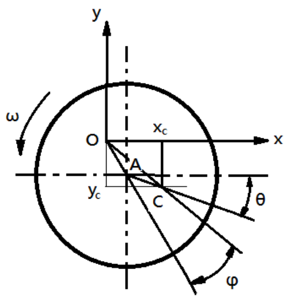

Fig. 1 Rotor eccentric model

2.1.2 Dynamic equation of single rotor system without squeeze film damper (SFD)

In the Fig. 1, C is the center of mass of the turntable, A is the center of the turntable, xc, yc are the coordinates of the center of mass of the turntable, CA is the mass eccentricity (CA=e), and ω is the rotation angular velocity. The relationship of the centroid coordinates of the turntable (x_c, y_c) and the fixed coordinates(x, y) is;

$$\begin{cases} x_c = x + e\cos\theta \\ y_c = y + e\sin\theta \end{cases} \tag{1}$$

Where $\theta = \omega t$

When the elastic force of the rotation axis is F, the motion equation of the center of mass is denoted as.

$$\begin{cases} m\ddot{x}_c = -kx \\ m\ddot{y}_c = -ky \end{cases} \tag{2}$$

Where

$$\begin{cases} \ddot{x}_c = \ddot{x} - e\omega^2 \sin \omega t \\ \ddot{y}_c = \ddot{y} - e\omega^2 \cos \omega t \end{cases} \tag{3}$$

By substituting equation (3) into equation (2), the motion differential equation of axial center C is obtained.

$$\begin{cases} \ddot{x} + \frac{k}{m}x = e\omega^2 \sin \omega t \\ \ddot{y} + \frac{k}{m}y = e\omega^2 \cos \omega t \end{cases} \tag{4}$$

If $\omega_n = \sqrt{\frac{k}{m}}$, the Eq. (4) can be written as follows.

$$\begin{cases} \ddot{x} + \omega_n^2 x = e\omega^2 \sin \omega t \\ \ddot{y} + \omega_n^2 y = e\omega^2 \cos \omega t \end{cases} \tag{5}$$

2.2 Dynamic modelling of single rotor system with squeeze film damper (SFD)

2.2.1 Model of single rotor system with squeeze film damper (SFD)

The rotor system model with SFD has two supports, the left support is an extruded film damper in parallel with an elastic support, the right support is a linear elastic support, and x_0 is the horizontal static eccentricity of the extruded film damper. y_0 is the static eccentricity in the vertical direction. In order to establish the motion equation of the system, four degrees of freedom are used to describe the motion characteristics of the system. The horizontal displacement of the center of mass of the rotor disc is x, the vertical displacement is y, the rotation angle about the horizontal direction is φ_x, and the rotation angle about the vertical direction is φ_y.

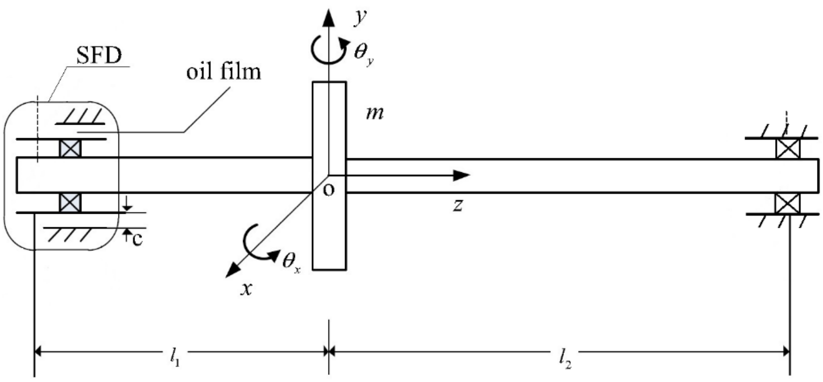

Fig.2 Single rotor system model with SFD

2.2.2 Dynamic equation of single rotor system with squeeze film damper (SFD)

2.2.2.1 The basic assumptions

The supporting form, disk parameters and shaft parameters of the rotor system are very important to the establishment of the rotor dynamic model. In order to facilitate the analysis, the following basic assumptions are made in the process of modeling.

1) The engine rotor system is simplified as a discrete structure, which consists of a disk with a concentrated mass, a rotating shaft and bearings without considering the mass.

2) The influence of axial vibration and torsional vibration is ignored, and only the bending vibration of the rotor is considered. It is assumed that the axial stiffness of the rotating shaft is large, and the axial deformation

of the rotor system is ignored. It is assumed that the torsional deformation of the rotating shaft is relatively small and can be ignored.

3) Suppose the disk is a rigid disk, and its unbalance is always in the same plane.

2.2.2.2 Dynamic equation of single rotor system with squeeze film damper (SFD)

①Kinetic energy of single rotor system

Since the mass of the rotating axis is not considered, the kinetic energy of the system is that of the disk:

$$T = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}J_d(\dot{\varphi}^2 + \dot{\varphi}^2) + \frac{1}{2}J_p\left(\omega^2 - 2\omega\dot{\varphi}_y\varphi_x\right) \tag{6}$$

Where J_p-The polar moment of inertia of a disk

J_d-The diameter moment of inertia of a disk

ω- The angular velocity of the disk

$J_p\omega\dot{\varphi}_y\varphi_x$-The energy produced by the gyro moment

②Elastic potential energy of single rotor system

Taking the static equilibrium position of the disk as the zero of potential energy, the potential energy of the system mainly includes the elastic potential energy of two elastic supports.

$$V_1 = \frac{1}{2}k_1\left[\left(x + l_1\varphi_y + x_0\right)^2 + (y - l_1\varphi_x - y_0)^2\right] \tag{7}$$

$$V_2 = \frac{1}{2}k_2\left[\left(x - l_2\varphi_y\right)^2 + (y + l_2\varphi_x)^2\right] \tag{8}$$

③Dissipated energy of single rotor system

Considering the linear damping, the dissipative energy of the system is as follows:

$$\psi_1 = \frac{1}{2}C_1\left[\left(\dot{x} + l_1\dot{\varphi}_y\right)^2 + (\dot{y} - l_1\dot{\varphi}_x)^2\right] \tag{9}$$

$$\psi_2 = \frac{1}{2}C_2\left[\left(\dot{x} - l_2\dot{\varphi}_y\right)^2 + (\dot{y} + l_2\dot{\varphi}_x)^2\right] \tag{10}$$

④Exciting force generated by unbalanced mass of single rotor system

When the center of gravity of the disc 0 does not coincide with the center of the rotating shaft $0'$, the excitation force caused by the unbalanced mass will be generated. Assuming that the unbalanced mass of the rotor system is concentrated on the disc and the distance from the axis center is e, then the excitation force of the rotor in the x and y directions is:

$$F_{e_x}(t) = me\omega^2 cos\omega t \tag{11}$$

$$F_{e_y}(t) = me\omega^2 sin\omega t \tag{12}$$

⑤Oil film force model

The basic formula of oil film force of squeeze film damper is the generalized Reynolds equation based on the oil film bearing.

$$\frac{\partial}{\partial X}\left(\frac{\rho h^3}{\mu}\cdot\frac{\partial p}{\partial X}\right) + \frac{\partial}{\partial Z}\left(\frac{\rho h^3}{\mu}\cdot\frac{\partial p}{\partial Z}\right) = 6(U_2 - U_1)\frac{\partial(\rho h)}{\partial X} + 12U_1 h\frac{\partial p}{\partial X} + 12p(V_1 - V_2) + 12h\frac{\partial p}{\partial t} \tag{13}$$

Where μ-Lubricating oil viscosity

h- Oil film thickness

p- Oil film pressure

$\partial h/\partial t = V_1 - V_2$ obtains the generalized Reynolds equation of compressible fluid with variable viscosity:

$$\frac{\partial}{\partial X}\left(\frac{\rho h^3}{\mu}\cdot\frac{\partial p}{\partial X}\right) + \frac{\partial}{\partial Z}\left(\frac{\rho h^3}{\mu}\cdot\frac{\partial p}{\partial Z}\right) = 6(U_2 - U_1)\frac{\partial(\rho h)}{\partial X} + 12\frac{\partial(\rho h)}{\partial t} + 12U_1 h\frac{\partial p}{\partial X} \tag{14}$$

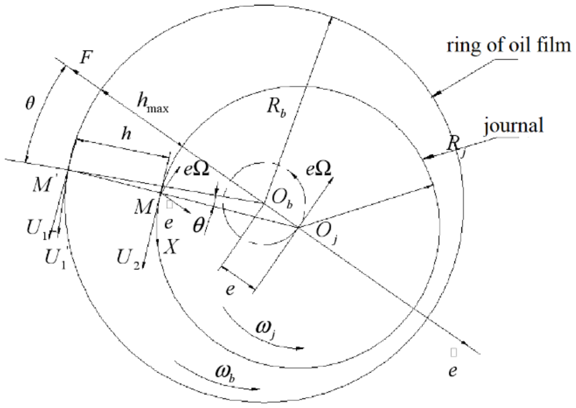

Fig. 3 SFD rotating coordinate system

Establish the SFD cylindrical coordinate system, as shown in Fig. 3, it is easy to know the oil film thickness h as:

$$h = R_b\sqrt{1-\frac{e^2}{R_b^2}sin^2\theta} - R_j + ecos\theta \tag{15}$$

Since e/R_b is far less than 1, and $R_b - R_j = c$, the eccentricity is defined as $\varepsilon = e/c$, so the film thickness h is:

$$h = c(1-\varepsilon\cos\theta) \tag{16}$$

According to the cylindrical coordinate system, $X = R_j\theta$ is obtained.

$$\frac{1}{R_j^2}\frac{\partial}{\partial\theta}\left(\frac{\rho h^3}{\mu}\frac{\partial p}{\partial X}\right) + \frac{\partial}{\partial Z}\left(\frac{\rho h^3}{\mu}\frac{\partial p}{\partial Z}\right) = 6\left(R_j\omega_b + R_j\omega_j + \dot{e}sin\theta - e\Omega cos\theta\right)\frac{1}{R_j^2}\frac{\partial(\rho h)}{\partial\theta} + 12\rho\left(\frac{\partial h}{\partial t} + e\Omega\sin\theta\right) + 12h\frac{\partial\rho}{\partial\theta} \tag{17}$$

Since C /R_j and e /R_j are far less than 1, and the radius of the oil film ring is very close to the journal, R_j=R, the oil film pressure is considered to be constant along the thickness of the oil film, without considering the rotation of the oil film ring and the journal. According to Eq. (16), the SFD Reynolds equation can be obtained as follows:

$$\frac{1}{R_j^2}\frac{\partial}{\partial\theta}\left(h^3\frac{\partial p}{\partial\theta}\right) + \frac{\partial}{\partial Z}\left(h^3\frac{\partial p}{\partial Z}\right) = -12\Omega\mu\frac{\partial h}{\partial\theta} \tag{18}$$

According to SFD short bearing theory, the change of oil film pressure along the axial Z direction is much greater than that along the circumference direction, so the $\partial p/\partial\theta$ term can be omitted. When the aspect ratio is less than 0.25, the oil enters the oil film clearance and tends to flow to both ends, so $\partial p/\partial Z$ has a great change. According to Eq. (18), it can be concluded that:

$$\frac{\partial}{\partial Z}\left(h^3\frac{\partial p}{\partial Z}\right) = -12\Omega\mu\frac{\partial h}{\partial\theta} \tag{19}$$

Substitute $\frac{\partial h}{\partial\theta} = -c\varepsilon sin\theta$ into Eq. (19) is as follows:

$$\frac{\partial}{\partial Z}\left(h^3\frac{\partial p}{\partial Z}\right) = 12\Omega\mu c\varepsilon sin\theta \tag{20}$$

When Z=±L/2 and p(θ,Z)=0, the approximate solution of SFD short bearing Reynolds equation can be obtained by taking the boundary conditions into the equation as follows:

$$p(\theta, Z) = \frac{6\mu\Omega c\varepsilon sin\theta}{h^3}\left(\frac{L^2}{4} - Z^2\right) \tag{21}$$

According to the half oil film in the positive pressure zone, the radial force of the oil film with cavities is calculated as:

$$F_r = \int_{-L/2}^{L/2}\int_{\pi}^{2\pi} p(\theta, Z)\, Rd\theta dZ \cdot cos\theta \tag{22}$$

According to the Z integral of Eq. (16) and Eq. (21) against Eq. (22), the tangential force of oil film can be obtained:

$$F_r = -\frac{\mu\Omega RL^3}{C^2}\int_{\pi}^{2\pi}\frac{\varepsilon sin\theta cos\theta}{(1+\varepsilon cos\theta)^3}d\theta \tag{23}$$

The theoretical oil film radial force of short bearing with holes is obtained by integration:

$$F_r = -\frac{\mu\Omega RL^3}{C^2}\left[\frac{2\varepsilon^2}{(1-\varepsilon^2)^2}\right] \tag{24}$$

Similarly, tangential force of oil film can be obtained as follows:

$$F_t = -\frac{\mu\Omega RL^3}{C^2}\int_{\pi}^{2\pi}\frac{\varepsilon sin\theta sin\theta}{(1+\varepsilon cos\theta)^3}d\theta \tag{25}$$

The theoretical oil film tangential force of short bearing with holes is obtained by integration:

$$F_t = -\frac{\mu\Omega RL^3}{C^2}\left[\frac{\pi\varepsilon}{2(1-\varepsilon^2)^{3/2}}\right] \tag{26}$$

The oil film force in the rectangular coordinate system can be expressed as:

$$\begin{cases} F_x = F_r\frac{x+l_1\varphi_y+x_0}{r} - F_t\frac{y-l_1\varphi_x-y_0}{r} \\ F_y = F_r\frac{y-l_1\varphi_x-y_0}{r} + F_t\frac{x+l_1\varphi_y+x_0}{r} \end{cases} \tag{27}$$

⑥Vibration differential equation of single rotor system with squeeze film damper (SFD)

Based on rotor dynamics theory and Lagrange method, the vibration differential equation of the system is established

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} + \frac{\partial \Psi_q}{\partial \dot{q}_j} = Q_j \tag{28}$$

Where L=T-V

L is Lagrange function of the system, T is the kinetic energy of the whole system, V is the potential energy of the system, q is the generalized coordinates, Ψ_q is the dissipated potential energy of the system, and Q_j is the generalized forces in the particle system other than the forces and viscous friction.

The vibration equation of the system established according to Eq. (28) is as follows:

$$m\ddot{x} + c_1(\dot{x} + l_1\dot{\varphi}_y) + K_1(x + l_1\varphi_y + x_0) + k_2(x - l_2\varphi_y) + F_x = me\omega^2 cos\omega t \tag{29}$$

$$m\ddot{y} + c_1(\dot{y} - l_1\dot{\varphi}_x) + c_2(\dot{y} + l_2\dot{\varphi}_x) + K_1(y - l_1\varphi_x - y_0) + k_2(y + l_2\varphi_x) + F_y = me\omega^2 sin\omega t \tag{30}$$

$$J_d\ddot{\varphi}_x + J_p\omega\dot{\varphi}_y + c_1l_1(l_1\dot{\varphi}_x - \dot{y}) + c_2l_2(l_2\dot{\varphi}_x + \dot{y}) + k_1l_1(l_1\varphi_x - y + y_0) + k_2l_2(l_2\varphi_x + y) - F_yl_1 = 0 \tag{31}$$

$$J_d\ddot{\varphi}_y - J_p\omega\dot{\varphi}_x + c_1l_1(l_1\dot{\varphi}_y + \dot{x}) + c_2l_2(l_2\dot{\varphi}_y - \dot{x}) + k_1l_1(l_1\varphi_y + x + x_0) + k_2l_2(l_2\varphi_y - x) + F_xl_1 = 0 \tag{32}$$

III. Dynamic modelling of dual rotor system

3.1 Dynamic modelling of the dual rotor system without squeeze film damper (SFD)

3.1.1 Dynamic model of the dual rotor system without squeeze film damper (SFD)

In the actual double-rotor structure, the mechanical model of the rotor system is simplified according to the characteristics of the asymmetrical distribution of the turntable and the different support modes of the high-low pressure rotors. In the process of simplification, the influence of high and low pressure rotor support asymmetry, mass eccentricity and gyro effect are considered. In the model, each support is simplified as a spring damping system.

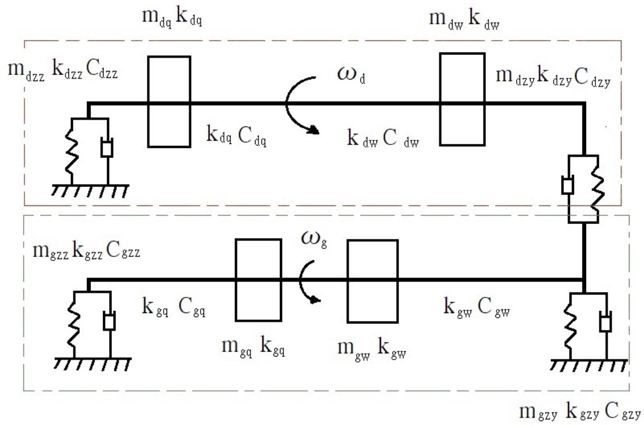

Fig.4 Mechanical model of dual rotor system

3.1.2 Dynamic modelling of dual rotor system without squeeze film damper (SFD)

3.1.2.1 Dynamic modelling of a low-pressure rotor system without squeeze film damper (SFD)

~ Parameters definition

In Fig. 4, m_{dzz}, m_{dzy}, m_{dq}, m_{dw} are the equivalent mass of the low-pressure rotor support, the low-pressure compressor, the low-pressure turbine disc and the low-pressure rotor support.

k_{dzz}, k_{dzy}, k_{dq}, k_{dw} are the equivalent stiffness of low pressure rotor support, low pressure compressor, low pressure turbine disc and low pressure rotor support.

C_{dzz}, C_{dzy}, C_{dq}, C_{dw} are equivalent damping of low pressure rotor support, low pressure compressor, low pressure turbine disc and low pressure rotor support.

J_{dzz}, J_{dzy}, J_{dq}, J_{dw} are the diameter moment of inertia and polar moment of inertia of the low-pressure compressor and low-pressure turbine disk.

μ_{dq}, μ_{dw} are the eccentricity of low-pressure compressor and low-pressure turbine disc.

ω_d, l_d are the speed and shaft length of the low-pressure rotor.

P_{dqx}, P_{dqy}, P_{dwx}, P_{dwy} are the external forces in X and Y directions of the low pressure compressor and the low pressure turbine disc.

F_{irmx}, F_{irmy} are the reaction forces of the intermediate bearing.

~ Dynamic modelling of a low-pressure rotor system

The kinetic energy, potential energy and dissipation functions of low-pressure compressor, low-pressure turbine disc and bearing in the Fig. 4 are:

$$T_{dq} = \frac{1}{2} m_{dq} \left[\left(\dot{x}_{dq} - u_{dq}\omega_d \sin \omega_d t \right)^2 + \left(\dot{y}_{dq} - u_{dq}\omega_d \cos \omega_d t \right) \right] + \frac{1}{2} \left[J_{dqd} \left(\dot{\theta}_{dx}^2 + \dot{\theta}_{dy}^2 \right) + J_{dqd}\omega^2 - 2J_{dqp}\omega\dot{\theta}_{dy}\theta_{dx} \right] \quad (33)$$

$$T_{dw} = \frac{1}{2} m_{dw} [(\dot{x}_{dw} - u_{dw}\omega_d \sin \omega_d t)^2 + (\dot{y}_{dw} - u_{dw}\omega_d \cos \omega_d t)] + \frac{1}{2} \left[J_{dwd} \left(\dot{\theta}_{dx}^2 + \dot{\theta}_{dy}^2 \right) + J_{dwd}\omega^2 - 2J_{dwp}\omega\dot{\theta}_{dy}\theta_{dx} \right] \quad (34)$$

$$T_{dz} = \frac{1}{2} m_{dzz} \left(\dot{x}_{dzz}^2 + \dot{y}_{dzz}^2 \right) + \frac{1}{2} m_{dzy} \left(\dot{x}_{dzy}^2 + \dot{y}_{dzy}^2 \right) \quad (35)$$

$$U_{dq} = \frac{1}{2} k_{dq} \left\{ \left(x_{dq} - \left(x_{dzz} + x_{dzy} \right)/2 \right)^2 + \left(y_{dq} - \left(y_{dzz} + y_{dzy} \right)/2 \right)^2 \right\} \quad (36)$$

$$U_{dw} = \frac{1}{2}k_{dw}\left\{\left(x_{dw} - \left(x_{dzz} + x_{dzy}\right)/2\right)^2 + \left(y_{dw} - \left(y_{dzz} + y_{dzy}\right)/2\right)^2\right\} \quad (37)$$

$$U_{dz} = \frac{1}{2}k_{dzz}\left(x_{dzz}^2 + y_{dzz}^2\right) + \frac{1}{2}k_{dzy}\left(x_{dzy}^2 + y_{dzy}^2\right) \quad (38)$$

$$D_{dq} = \frac{1}{2}c_{dq}\left\{\left[\dot{x}_{dq} - \left(\dot{x}_{dzz} + \dot{x}_{dzy}\right)/2\right]^2 + \left[\dot{y}_{dq} - \left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)/2\right]^2\right\} \quad (39)$$

$$D_{dw} = \frac{1}{2}c_{dw}\left\{\left[\dot{x}_{dw} - \left(\dot{x}_{dzz} + \dot{x}_{dzy}\right)/2\right]^2 + \left[\dot{y}_{dw} - \left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)/2\right]^2\right\} \quad (40)$$

$$D_{dz} = \frac{1}{2}c_{dzz}\left(\dot{x}_{dzz}^2 + \dot{y}_{dzz}^2\right) + \frac{1}{2}c_{dzy}\left(\dot{x}_{dzy}^2 + \dot{y}_{dzz}^2\right) \quad (41)$$

Where

$$\theta_{dx} = \frac{y_{dzz} - y_{dzy}}{l_d}, \theta_{dy} = \frac{x_{dzz} - x_{dzy}}{l_d} \quad (42)$$

According to the Lagrange equation of non-conservative systems:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = Q_j \quad (43)$$

The differential equation of the low-pressure rotor system is obtained as follows:

$$m_{dq}\ddot{x}_{dq} + k_{dq}\left[x_{dq} - \left(x_{dzz} + x_{dzy}\right)/2\right] + C_{dq}\left[\dot{x}_{dq} - \left(\dot{x}_{dzz} + \dot{x}_{dzy}\right)/2\right] = m_{dq}u_{dq}\omega_d^2 \cos\omega t + P_{dqx} \quad (44)$$

$$m_{dq}\ddot{y}_{dq} + k_{dq}\left[y_{dq} - \left(y_{dzz} + y_{dzy}\right)/2\right] + C_{dq}\left[\dot{y}_{dq} - \left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)/2\right] = m_{dq}u_{dq}\omega_d^2 \sin\omega t + P_{dqy} - m_{dq}\, g \quad (45)$$

$$m_{dw}\ddot{x}_{dw} + k_{dw}\left[x_{dw} - \left(x_{dzz} + x_{dzy}\right)/2\right] + C_{dw}\left[\dot{x}_{dw} - \left(\dot{x}_{dzz} + \dot{x}_{dzy}\right)/2\right] = m_{dw}u_{dw}\omega_d^2 \cos\omega t + P_{dwx} \quad (46)$$

$$m_{dw}\ddot{y}_{dw} + k_{dw}\left[y_{dw} - \left(y_{dzz} + y_{dzy}\right)/2\right] + C_{dw}\left[\dot{y}_{dq} - \left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)/2\right] = m_{dw}u_{dw}\omega_d^2 \sin\omega t + P_{dwy} - m_{dw}g \quad (47)$$

$$m_{dzz}\ddot{x}_{dzz} + k_{dzz}x_{dzz} + C_{dzz}\dot{x}_{dzz} + J_{dq}\frac{\ddot{x}_{dzy} - \ddot{x}_{dzz}}{l_d^2} + J_{dw}\frac{\ddot{x}_{dzy} - \ddot{x}_{dzz}}{l_d^2} - J_{dq}\omega_d\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} - J_{dw}\omega_d\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} - \frac{1}{2}k_{dq}\left[x_{dq} - \frac{\left(x_{dzz} + x_{dzy}\right)}{2}\right] - \frac{1}{2}k_{dw}\left[x_{dw} - \frac{\left(x_{dzz} + x_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dq}\left[\dot{y}_{dq} - \frac{\left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dw}\left[\dot{y}_{dw} - \frac{\left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)}{2}\right] = 0 \quad (48)$$

$$m_{dzz}\ddot{y}_{dzz} + k_{dzz}y_{dzz} + C_{dzz}\dot{y}_{dzz} + J_{dq}\frac{\ddot{y}_{dzy} - \ddot{y}_{dzz}}{l_d^2} + J_{dw}\frac{\ddot{y}_{dzy} - \ddot{y}_{dzz}}{l_d^2} - J_{dq}\omega\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} - J_{dw}\omega\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} - \frac{1}{2}k_{dq}\left[y_{dq} - \frac{\left(y_{dzz} + y_{dzy}\right)}{2}\right] - \frac{1}{2}k_{dw}\left[y_{dw} - \frac{\left(y_{dzz} + y_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dq}\left[\dot{x}_{dq} - \frac{\left(\dot{x}_{dzz} + \dot{x}_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dw}\left[\dot{x}_{dw} - \frac{\left(\dot{x}_{dzz} + \dot{x}_{dzy}\right)}{2}\right] = -m_{dzz}g \quad (49)$$

$$m_{dzy}\ddot{x}_{dzy} + k_{dzy}x_{dzy} + C_{dzy}\dot{x}_{dzy} + J_{dq}\frac{\ddot{x}_{dzy} - \ddot{x}_{dzz}}{l_d^2} + J_{dw}\frac{\ddot{x}_{dzy} - \ddot{x}_{dzz}}{l_d^2} + J_{dq}\omega\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} + J_{dw}\omega\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} - \frac{1}{2}k_{dq}\left[x_{dq} - \frac{\left(x_{dzz} + x_{dzy}\right)}{2}\right] - \frac{1}{2}k_{dw}\left[x_{dw} - \frac{\left(x_{dzz} + x_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dq}\left[\dot{y}_{dq} - \frac{\left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dw}\left[\dot{y}_{dw} - \frac{\left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)}{2}\right] = -F_{irmx} \quad (50)$$

$$m_{dzy}\ddot{y}_{dzy} + k_{dzy}y_{dzy} + C_{dzy}\dot{y}_{dzy} + J_{dq}\frac{\ddot{y}_{dzy} - \ddot{y}_{dzz}}{l_d^2} + J_{dw}\frac{\ddot{y}_{dzy} - \ddot{y}_{dzz}}{l_d^2} + J_{dq}\omega\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} + J_{dw}\omega\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} - \frac{1}{2}k_{dq}\left[y_{dq} - \frac{(y_{dzz} + y_{dzy})}{2}\right] - \frac{1}{2}k_{dw}\left[y_{dw} - \frac{(y_{dzz} + y_{dzy})}{2}\right] - \frac{1}{2}C_{dq}\left[\dot{x}_{dq} - \frac{(\dot{x}_{dzz} + \dot{x}_{dzy})}{2}\right] - \frac{1}{2}C_{dw}\left[\dot{x}_{dw} - \frac{(\dot{x}_{dzz} + \dot{x}_{dzy})}{2}\right] = F_{irmy} - m_{dzy}g \quad (51)$$

3.1.2.2 Dynamic modelling of a high-pressure rotor system

˜ Parameters definition

In Fig. 4, $m_{gzz}, m_{gzy}, m_{gq}, m_{gw}$ are the equivalent mass of the high-pressure rotor support, the high-pressure compressor, the high-pressure turbine disc and the high-pressure rotor support.

$k_{gzz}, k_{gzy}, k_{gq}, k_{gw}$ are the equivalent stiffness of high pressure rotor support, high pressure compressor, high pressure turbine disc and high pressure rotor support.

$C_{gzz}, C_{gzy}, C_{gq}, C_{gw}$ are equivalent damping of high pressure rotor support, high pressure compressor, high pressure turbine disc and high pressure rotor support.

$J_{gpd}, J_{gpp}, J_{gwd}, J_{gwp}$ are the diameter moment of inertia and polar moment of inertia of the high - pressure compressor and high-pressure turbine disk.

μ_{gq}, μ_{gw} are the eccentricity of high -pressure compressor and high -pressure turbine disc.

ω_g, l_g are the speed and shaft length of the high-pressure rotor.

$P_{gqx}, P_{gqy}, P_{gwx}, P_{gwy}$ are the external forces in X and Y directions of the high pressure compressor and the high pressure turbine disc.

F_{irmx}, F_{irmy} are the reaction forces of the intermediate bearing.

˜ Dynamic modelling of a high-pressure rotor system

The kinetic energy, potential energy and dissipation functions of high -pressure compressor, high -pressure turbine disc and bearing in the Fig. 4 are:

$$T_{gq} = \frac{1}{2}m_{gq}\left[(\dot{x}_{gq} - u_{gq}\omega_g \sin\omega_g t)^2 + (\dot{y}_{gq} - u_{gq}\omega_g \cos\omega_g t)\right] + \frac{1}{2}\left[J_{gqd}(\dot{\theta}_{gx}^2 + \dot{\theta}_{gy}^2) + J_{gqd}\omega^2 - 2J_{gqp}\omega\dot{\theta}_{gy}\theta_{gx}\right] \quad (52)$$

$$T_{gw} = \frac{1}{2}m_{gw}\left[(\dot{x}_{gw} - u_{gw}\omega_g \sin\omega_g t)^2 + (\dot{y}_{gw} - u_{gw}\omega_g \cos\omega_g t)\right] + \frac{1}{2}\left[J_{gwd}(\dot{\theta}_{gx}^2 + \dot{\theta}_{gy}^2) + J_{gwd}\omega^2 - 2J_{gwp}\omega\dot{\theta}_{gy}\theta_{gx}\right] \quad (53)$$

$$T_{gz} = \frac{1}{2}m_{gzz}(\dot{x}_{gzz}^2 + \dot{y}_{gzz}^2) + \frac{1}{2}m_{gzy}(\dot{x}_{gzy}^2 + \dot{y}_{gzy}^2) \quad (54)$$

$$U_{gq} = \frac{1}{2}k_{gq}\left\{(x_{gq} - (x_{gzz} + x_{gzy})/2)^2 + (y_{gq} - (y_{gzz} + y_{gzy})/2)^2\right\} \quad (55)$$

$$U_{gw} = \frac{1}{2}k_{gw}\left\{(x_{gw} - (x_{gzz} + x_{gzy})/2)^2 + (y_{gw} - (y_{gzz} + y_{gzy})/2)^2\right\} \quad (56)$$

$$U_{gz} = \frac{1}{2}k_{gzz}(x_{gzz}^2 + y_{gzz}^2) + \frac{1}{2}k_{gzy}(x_{gzy}^2 + y_{gzy}^2) \quad (57)$$

$$D_{gq} = \frac{1}{2}c_{gq}\left\{[\dot{x}_{gq} - (\dot{x}_{gzz} + \dot{x}_{gzy})/2]^2 + [\dot{y}_{gq} - (\dot{y}_{gzz} + \dot{y}_{gzy})/2]^2\right\} \quad (58)$$

$$D_{gw} = \frac{1}{2}c_{gw}\left\{[\dot{x}_{gw} - (\dot{x}_{gzz} + \dot{x}_{gzy})/2]^2 + [\dot{y}_{gw} - (\dot{y}_{gzz} + \dot{y}_{gzy})/2]^2\right\} \quad (59)$$

$$D_{gz} = \frac{1}{2}c_{gzz}(\dot{x}_{gzz}^2 + \dot{y}_{gzz}^2) + \frac{1}{2}c_{gzy}(\dot{x}_{gzy}^2 + \dot{y}_{gzz}^2) \quad (60)$$

Where

$$\theta_{gx} = \frac{y_{gzz}-y_{gzy}}{l_g}, \theta_{gy} = \frac{x_{gzz}-x_{gzy}}{l_g} \quad (61)$$

According to the Lagrange equation of non-conservative systems:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = Q_j \quad (62)$$

The differential equation of the high-pressure rotor system is obtained as follows:

$$m_{gq}\ddot{x}_{gq} + k_{gq}[x_{gq} - (x_{gzz} + x_{gzy})/2] + C_{gq}[\dot{x}_{gq} - (\dot{x}_{gzz} + \dot{x}_{gzy})/2] = m_{gq}u_{gq}\omega_g^2 \cos\omega t + P_{gqx} \quad (63)$$

$$m_{gq}\ddot{y}_{gq} + k_{gq}[y_{gq} - (y_{gzz} + y_{gzy})/2] + C_{gq}[\dot{y}_{gq} - (\dot{y}_{gzz} + \dot{y}_{gzy})/2] = m_{gq}u_{gq}\omega_g^2 \sin\omega t + P_{gqy} - m_{gq}\, g \quad (64)$$

$$m_{gw}\ddot{x}_{gw} + k_{gw}[x_{gw} - (x_{gzz} + x_{gzy})/2] + C_{gw}[\dot{x}_{gw} - (\dot{x}_{gzz} + \dot{x}_{gzy})/2] = m_{gw}u_{gw}\omega_g^2 \cos\omega t + P_{gwx} \quad (65)$$

$$m_{gw}\ddot{y}_{gw} + k_{gw}[y_{gw} - (y_{gzz} + y_{gzy})/2] + C_{gw}[\dot{y}_{gq} - (\dot{y}_{gzz} + \dot{y}_{gzy})/2] = m_{gw}u_{gw}\omega_g^2 \sin\omega t + P_{gwy} - m_{gw}g \quad (66)$$

$$m_{gzz}\ddot{x}_{gzz} + k_{gzz}x_{gzz} + C_{gzz}\dot{x}_{gzz} + J_{gq}\frac{\ddot{x}_{gzy} - \ddot{x}_{gzz}}{l_g^2} + J_{gw}\frac{\ddot{x}_{gzy} - \ddot{x}_{gzz}}{l_g^2} - J_{dq}\omega_g\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} - J_{gw}\omega_g\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} - \frac{1}{2}k_{gq}\left[x_{gq} - \frac{(x_{gzz} + x_{gzy})}{2}\right] - \frac{1}{2}k_{gw}\left[x_{gw} - \frac{(x_{gzz} + x_{gzy})}{2}\right] - \frac{1}{2}C_{gq}\left[\dot{y}_{gq} - \frac{(\dot{y}_{gzz} + \dot{y}_{gzy})}{2}\right] - \frac{1}{2}C_{gw}\left[\dot{y}_{gw} - \frac{(\dot{y}_{gzz} + \dot{y}_{gzy})}{2}\right] = 0 \quad (67)$$

$$m_{gzz}\ddot{y}_{gzz} + k_{gzz}y_{gzz} + C_{gzz}\dot{y}_{gzz} + J_{gq}\frac{\ddot{y}_{gzy} - \ddot{y}_{gzz}}{l_g^2} + J_{gw}\frac{\ddot{y}_{gzy} - \ddot{y}_{gzz}}{l_g^2} - J_{gq}\omega\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} - J_{gw}\omega\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} - \frac{1}{2}k_{gq}\left[y_{gq} - \frac{(y_{gzz} + y_{gzy})}{2}\right] - \frac{1}{2}k_{gw}\left[y_{gw} - \frac{(y_{gzz} + y_{gzy})}{2}\right] - \frac{1}{2}C_{gq}\left[\dot{x}_{gq} - \frac{(\dot{x}_{gzz} + \dot{x}_{gzy})}{2}\right] - \frac{1}{2}C_{gw}\left[\dot{x}_{gw} - \frac{(\dot{x}_{gzz} + \dot{x}_{gzy})}{2}\right] = -m_{gzz}g \quad (68)$$

$$m_{gzy}\ddot{x}_{gzy} + k_{gzy}x_{gzy} + C_{gzy}\dot{x}_{gzy} + J_{gq}\frac{\ddot{x}_{gzy} - \ddot{x}_{gzz}}{l_g^2} + J_{gw}\frac{\ddot{x}_{gzy} - \ddot{x}_{gzz}}{l_g^2} + J_{gq}\omega\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} + J_{gw}\omega\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} - \frac{1}{2}k_{gq}\left[x_{gq} - \frac{(x_{gzz} + x_{gzy})}{2}\right] - \frac{1}{2}k_{gw}\left[x_{gw} - \frac{(x_{gzz} + x_{gzy})}{2}\right] - \frac{1}{2}C_{gq}\left[\dot{y}_{gq} - \frac{(\dot{y}_{gzz} + \dot{y}_{gzy})}{2}\right] - \frac{1}{2}C_{gw}\left[\dot{y}_{gw} - \frac{(\dot{y}_{gzz} + \dot{y}_{gzy})}{2}\right] = -F_{irmx} \quad (69)$$

$$m_{gzy}\ddot{y}_{gzy} + k_{gzy}y_{gzy} + C_{gzy}\dot{y}_{gzy} + J_{gq}\frac{\ddot{y}_{gzy} - \ddot{y}_{gzz}}{l_g^2} + J_{gw}\frac{\ddot{y}_{gzy} - \ddot{y}_{gzz}}{l_g^2} + J_{gq}\omega\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} + J_{gw}\omega\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} - \frac{1}{2}k_{gq}\left[y_{gq} - \frac{(y_{gzz} + y_{gzy})}{2}\right] - \frac{1}{2}k_{gw}\left[y_{gw} - \frac{(y_{gzz} + y_{gzy})}{2}\right] - \frac{1}{2}C_{gq}\left[\dot{x}_{gq} - \frac{(\dot{x}_{gzz} + \dot{x}_{gzy})}{2}\right] - \frac{1}{2}C_{gw}\left[\dot{x}_{gw} - \frac{(\dot{x}_{gzz} + \dot{x}_{gzy})}{2}\right] = F_{irmy} - m_{gzy}g \quad (70)$$

3.2 Dynamic modelling of the dual rotor system with squeeze film damper (SFD)

3.2.1 Dynamic model of the dual rotor system with squeeze film damper (SFD)

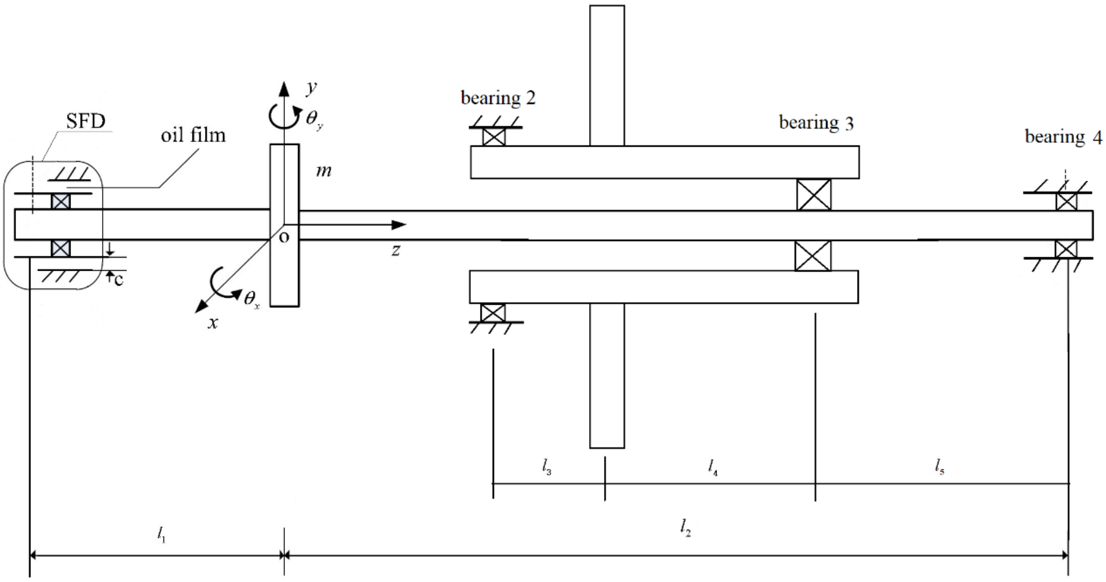

Fig.5 Schematic diagram of dual rotor system model

3.2.2 Dynamic modelling of the dual rotor system with squeeze film damper (SFD)

The wheels of the high and low pressure rotors of the dual-rotor system are simplified into single wheels. The outer rotor simulates the high pressure rotor of an aeroengine, the inner rotor simulates the low pressure rotor, and the left end support of the inner rotor is an squeeze oil film damper in parallel with a centering elastic support, and the shaft is simplified for an annular hollow beam with a uniform cross section, the system has a total of 4 supports, including elastic supports A, B and C of 3 bearing seats, and an intermediate support D connecting the high-pressure rotor and the low-pressure rotor. The movement of the system is described by 8 degrees of freedom: The horizontal displacement of the center of mass of the low-pressure rotor disc is x_1, the vertical displacement is y1, the rotation angle θ_x around the horizontal direction, the rotation angle θ_y around the vertical direction; the horizontal displacement of the centroid of the high-pressure rotor disc x_2, the vertical displacement y_2, the rotation angle around the horizontal direction φ_x, the turning angle φ_y around the vertical direction.

① Nonlinear bearing forces of intermediate bearings

The intermediate bearing is a device that supports the high and low pressure rotors. When the radial load acts on the rolling bearing, its rolling elements pass through the load zone in turn during operation. Each time the rolling elements pass the load line of action, a variable stiffness will be generated. Vibration, the frequency of variable stiffness is related to the rotation angular velocity of the cage and the number of rolling elements, which can be expressed as:

$$f_{vc} = \frac{\omega_m N_b}{2\pi} \quad (71)$$

Where f_{vc} is the frequency of bearing variable stiffness vibration, ω_m is the rotation angular speed of cage, N_b is the number of rolling elements.

The motion diagram of the intermediate bearing is shown in Fig.8. As can be seen from the Fig.8, the linear velocity at the contact point between the rolling element and the inner ring is V_i, and the linear velocity at the contact point between the rolling element and the outer ring is V_0, and the relation is as follows:

$V_i = \omega_i r_i$

$$V_0 = \omega_0 r_0 \tag{72}$$

Where ω_i and ω_0 is the rolling angular velocity of the inner and outer rings of the bearing, r_i and r_0 is the radius of the inner and outer rings of the bearing.

Assuming that there is pure rolling between the rolling elements and the inner and outer rings, the linear speed of rotation of the cage V_m is equal to the linear speed of revolution of the rolling elements, which can be obtained:

$$V_m = \frac{V_i + V_0}{2} \tag{73}$$

From Eq. (72) to Eq. (73), the revolution angular velocity of the rolling element, ω_b can be obtained as:

$$\omega_b = \frac{\omega_i r_i + \omega_0 r_0}{(r_i + r_0)} \tag{74}$$

According to Fig. 6, the rotation Angle θ_j of the j-th roller at time t is as follows:

$$\theta_j = \frac{2\pi}{N_b}(j-1) + \omega_b t, \ (j=1,2,\ldots N_b) \tag{75}$$

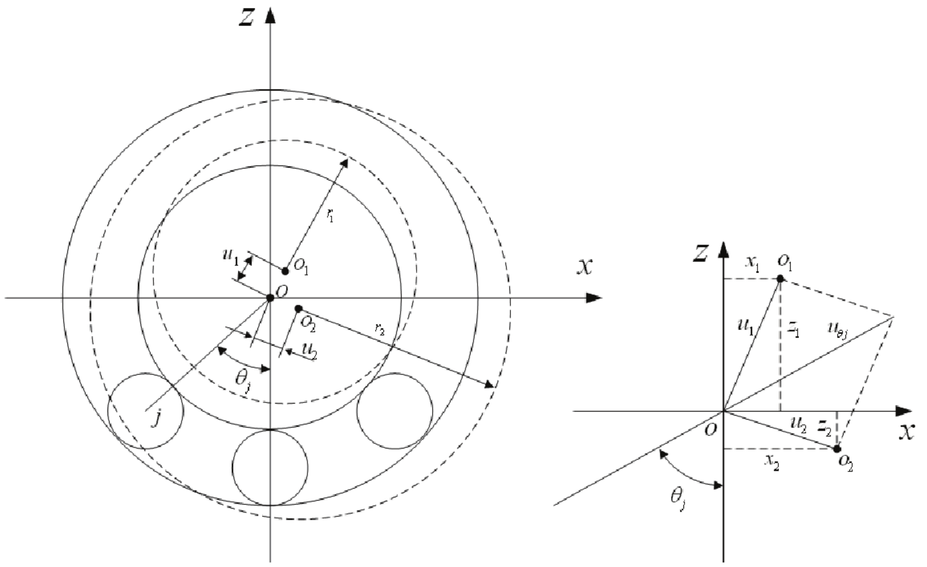

Fig.6 Schematic diagram of position relationship between inner and outer ring of intermediate bearing

The radial elastic contact deformation of the i-th ball is:

$$\delta_i = (x_i - x_0)cos\theta_i + (y_i - y_0)sin\theta_i - \delta_0 \tag{76}$$

Where x_i , x_0, y_i, y_0 are respectively the horizontal and vertical displacement components of the inner and outer rings of the bearing, δ_0 is the radial clearance of the bearing.

$x_i = x_1 - (l_2 - l_5)\theta_y$

$x_0 = x_2 - l_4\varphi_y$

$y_i = y_1 + (l_2 - l_5)\theta_x$

$$y_0 = y_2 + l_4\varphi_x \tag{77}$$

According to the Hertz contact theorem, the relationship between the radial contact force and deformation of the rolling element can be expressed as [12]:

$$\begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = C_b \sum_{i=1}^{N_b} (\delta_i G(\delta_i))^n \begin{bmatrix} cos\theta_i \\ sin\theta_i \end{bmatrix}, \text{(i=1,2,...}N_b) \tag{78}$$

Where C_b is Hertz contact stiffness of the bearing.

$$G(\delta_i) = \begin{cases} 1 & \delta_i > 0 \\ 0 & \delta_i \le 0 \end{cases} \tag{79}$$

For ball bearings, n takes 3/2, for cylindrical roller bearings, n takes 10/9.

② Short bearing π oil film model

Using Eq. (27) oil film force model is as follows:

$$\begin{cases} F_x = F_r \frac{x+l_1\theta_y}{r} - F_t \frac{y-l_1\theta_x}{r} \\ F_y = F_r \frac{y-l_1\varphi_x}{r} + F_t \frac{x+l_1\varphi_y}{r} \end{cases} \tag{80}$$

③ Vibration differential equation of dual rotor system with squeeze film damper (SFD)

The vibration equation of the system established according to Eq. (28) is as follows:

$$m_1\ddot{x}_1 + c_1(\dot{x}_1 + l_1\dot{\theta}_y) + c_2(\dot{x}_1 - l_2\dot{\theta}_y) + k_1(x_1 - l_1\theta_y) + k_2(x_1 - l_2\theta_y) + Q_x + F_x = m_1e_1\omega_1^2cos\omega_1 t \tag{81}$$

$$m_1\ddot{y}_1 + c_1(\dot{y}_1 - l_1\dot{\theta}_x) + c_2(\dot{y}_1 + l_2\dot{\theta}_x) + k_1(y_1 - l_1\theta_x) + k_2(y_1 + l_2\theta_x) + Q_y + F_y = m_1e_1\omega_1^2sin\omega_1 t - -m_1 g \tag{82}$$

$$m_2\ddot{x}_2 + c_3(\dot{x}_2 + l_3\dot{\varphi}_y) + k_3(x_2 + l_3\varphi_y) - Q_x = m_2e_2\omega_2^2cos\omega_2 t \tag{83}$$

$$m_2\ddot{y}_2 + c_3(\dot{y}_2 - l_3\dot{\varphi}_x) + k_3(y_2 - l_3\varphi_x) - Q_y = m_2e_2\omega_2^2sin\omega_2 t - m_2 g \tag{84}$$

$$J_{d_1}\ddot{\theta}_x + J_{p_1}\omega_1\dot{\theta}_y + c_1l_1(l_1\dot{\theta}_x - \dot{y}_1) + c_2l_2(l_2\dot{\theta}_x + \dot{y}_1) + k_1l_1(l_1\theta_x - y_1) + k_2l_2(l_2\theta_x + y_1) + Q_y(l_2 - l_5) - F_y l_1 = 0 \tag{85}$$

$$J_{d_1}\ddot{\theta}_y - J_{p_1}\omega_1\dot{\theta}_x + c_1l_1(l_1\dot{\theta}_y + \dot{x}_1) + c_2l_2(l_2\dot{\theta}_y - \dot{x}_1) + k_1l_1(l_1\theta_y + x_1) + k_2l_2(l_2\theta_y - x_1) - -Q_x(l_2 - l_5) + F_x l_1 = 0 \tag{86}$$

$$J_{d_2}\ddot{\varphi}_x + J_{p_2}\omega_2\dot{\varphi}_y + c_3l_3(l_3\dot{\varphi}_x - \dot{y}_2) + k_3l_3(l_3\varphi_x - y_2) - Q_y l_4 = 0 \tag{87}$$

$$J_{d_2}\ddot{\varphi}_y - J_{p_2}\omega_2\dot{\varphi}_x + c_3l_3(l_3\dot{\varphi}_y + \dot{x}_2) + k_3l_3(l_3\varphi_y + x_2) + Q_x l_4 = 0 \tag{88}$$

III. CONCLUSION

As the basic structure of the engine, the rotor system is the main source of engine vibration due to its unbalance characteristics and vibration in the critical speed region. Therefore, it has become a trend to equip the extruder film damper on the aero-engine. Generally, due to the influence of processing technology, structure dead weight, structure design and assembly technology, the extruded oil film damper itself has static eccentricity, which affects the stable operation of the system to a certain extent. Therefore, the theoretical research on the vibration characteristics of the extruded oil film damper rotor system has very important engineering value and practical significance. Therefore, in this dissertation, the task of research was to create a mathematical model of the rotor system with and without the squeeze film damper. First, the model of a single rotor system with and without the squeeze film damper was made. The modeling method of a simple single rotor system with one disk and two supports was specifically mentioned. Next, the mathematical modelling method of the dual rotor system with and without the squeeze film damper was mentioned. The dual rotor system is composed of a low-pressure rotor part and a high-pressure rotor part. Firstly, the mathematical model of low pressure rotor system of double rotor system was built. In modelling, the effects of asymmetry, mass eccentricity, and gyroscopic effect of the low-pressure rotor system and the squeeze film damper were fully considered and the differential equation of the low-pressure rotor system was written by utilizing Lagrange equation. Next, a differential equation of a high pressure rotor system was formulated, considering sufficient conditions such as low pressure rotor system.

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