

# Hydro Magnetic Flow Past an Infinite Vertical Oscillating Plate

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## ABSTRACT

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The present work is confined to obtain the numerical solution of an unsteady free convection flow of a viscous incompressible fluid past an infinite vertical oscillating plate. Magnetic field is applied normal to the flow. The governing equations are solved, numerically by implicit finite difference method. The obtained results are discussed and analyzed with the aid of graphs. The results obtained are in good agreement with realistic physical phenomenon.

**Keywords :** Magnetic field, Free convection, Oscillating plate, Implicit finite difference technique.

## I. INTRODUCTION

The Viscous Dissipation effect on Natural Convection is studied by Gebhart [1]. Nanda and Sharma [2] analyzed the free convection effects on oscillatory flow. Hellums and Churchill[3] discussed the finite difference method on natural convection. Nanda and Sharma [4] discussed the effects of free convection in oscillatory flow. Singh [5] studied the free convection flow past a vertical infinite flat plate in the presence of magnetic field. The exact solution of the Navier-Stokes equation was first given by Stokes [6] which is concerned with the flow of a viscous incompressible fluid past an infinite horizontal plate oscillating in its own plane in an infinite mass of stationary fluid. Siegel [7] studied transient free convection from a vertical flat plate. Natural convection in vicinity of a double infinite vertical plate is discussed by Schetz and Eichhorn [8]. Menold and Yang [9] studied an asymptotic solutions for free convection on a vertical plate. Transient free convection about plates and

circular cylinders is analyzed by Goldstein and Briggs [10].

Soundalgekar [11] studied the effects of suction and viscous dissipation on free-Convective flow of an incompressible fluid past an infinite vertical porous plate with mass transfer. Soundalgekar [12] studied free convection current effects on the flow past a vertical oscillating plate. Soundalgekar [13] discussed free convection flow past a vertical oscillating plate with constant heat flux, Soundalgekar et al [14] unsteady flow of an elastico-viscous fluid past an infinite vertical plate. Transient free convection past a semi-infinite vertical plate with variable surface temperature is studied by Takhar et al [15]. Soundalgekar et al [16] analyzed the effects viscous dissipation and transient free convection on flow of an incompressible fluid. Soundalgekar et al [17] studied free convection effects on flow past an infinite vertical oscillating plate with constant heat flux in the presence of magnetic field. Transient free convection flow past an infinite vertical plate with temperature gradient dependent heat source is discussed by

Soundalgekar and Jaiswal,[18]. Soundalgekar et al [19] studied the effects of periodic temperature variation on transient free convection flow past an infinite vertical plate. Sriramulu et al [20] analyzed the effects of applied magnetic field on transient free convective flow of an incompressible fluid in a vertical channel with viscous dissipation. Srihari et al. [21] discussed free convection flow an incompressible viscous dissipative fluid in an infinite vertical oscillating plate in the presence of magnetic field with constant heat flux. Anand rao and Srinivasa Raju, [22] reported the effects of Applied magnetic field on transient free convective flow of an incompressible fluid in a vertical channel with viscous dissipation. Srinivasa Rao and Anand Babu [23] obtained a finite element solution of free convection flow of an incompressible viscous dissipative fluid in an infinite vertical oscillating plate in the presence of magnetic field. Mishra et al [24] studied the mass transfer effect on hydro-magnetic flow of a visco-elastic fluid through porous medium with oscillatory suction and heat source. Jyotsna Rani Pattnaik et al [25] studied the radiation and mass transfer effects on flow through porous medium past an exponentially accelerated inclined plate in the presence of magnetic field. Raptis and Kafousias [26] studied free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate. Shankar Goud et al [27] made finite element analysis on an unsteady free convection flow of Casson fluid past a vertical oscillating plate in porous medium .

The present study is confined to obtain the numerical solution of free convection flow of a viscous incompressible fluid past an infinite vertical oscillating plate with constant heat flux under the influence of a transverse magnetic field. This problem is governed by coupled non-linear system of partial differential equations, for which an implicit finite difference method is employed to solve the governing equations of the motion.

## II. Mathematical Analysis

Consider the flow of an incompressible electrically conducting viscous fluid, which is initially at rest and surrounds an infinite vertical plate at temperature  $T_\infty'$ . The  $x'$  -axis is taken along the plate in the vertically upward direction and the  $y'$ -axis is taken normal to the plate. At time  $t' > 0$ , a uniform magnetic field is applied in the direction of the  $y'$ -axis and the plate starts oscillating in its own plane with heat supplied to the plate at constant rate.

Therefore, under these assumptions and usual Boussinesq's approximation, the MHD flow is governed by the following differential equations:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta (T' - T_\infty') - \frac{\sigma B_0^2}{\rho} u' \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p} k \frac{\partial^2 T'}{\partial y'^2} + \frac{Q(T - T_\infty)}{\rho c_p} \quad (2)$$

$$\frac{\partial C}{\partial t'} + v' \frac{\partial C}{\partial y'} = D \frac{\partial^2 C}{\partial y'^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y'^2} \quad (3)$$

with the following initial and boundary conditions :

$$\begin{aligned} u' &= 0 & T' &= T_\infty' & \text{for all } y' & t' \leq 0 \\ u' &= U_0 \cos \omega' t' & \frac{\partial T'}{\partial y'} &= -\frac{q'}{k} & \text{at } y' = 0 & t' > 0 \\ u' &\rightarrow 0; & T' &\rightarrow T_\infty' & \text{as } y' \rightarrow \infty \end{aligned} \quad (4)$$

Introducing the following non-dimensional quantities:

$$u = \frac{u'}{U_0}, \quad t = \frac{t'U_0^2}{\nu}, \quad y = \frac{y'U_0}{\nu}, \quad M = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, \quad P_r = \frac{\mu C_p}{k}, \quad Ec = \frac{kU_0^3}{C_p q \nu} \quad (5)$$

$$\omega = \frac{\omega' \nu}{U_0^2}, \quad \theta = \frac{T' - T_\infty'}{q \nu / k U_0}, \quad G = \frac{g \beta q \nu^2}{k U_0^4}, \quad So = \frac{D_m k_T (T_w - T_\infty)}{\nu T_m (C_w - C_\infty)}, \quad S = \frac{Q \nu}{\rho C_p U_0^2}$$

in equations (1) & (3), the following obtained

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G\theta + GmC - Mu \quad (6)$$

$$P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + S\theta \quad (7)$$

$$Sc \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} + SoSc \frac{\partial^2 \theta}{\partial y^2} \quad (8)$$

The initial and boundary conditions in dimension less form are:

$$\begin{aligned} u=0; \quad \theta=0 \quad \text{for all } y \quad t \leq 0 \\ u = \cos \omega t; \quad \frac{d\theta}{dy} = -1 \quad \text{at } y=0 \\ u \rightarrow 0; \quad \theta \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad (9)$$

### III. Method of Solution

Applying the implicit finite difference method on governing equations (6) and (8), the following system of equations are obtained.

$$-\frac{r}{2}u_{i-1}^{j+1} + (1+r)u_i^{j+1} - \frac{r}{2}u_{i+1}^{j+1} = A_i^j \quad (10)$$

$$-\frac{r}{2}\theta_{i-1}^{j+1} + (P_r+r)\theta_i^{j+1} - \frac{r}{2}\theta_{i+1}^{j+1} = B_i^j \quad (11)$$

$$-\frac{r}{2}C_{i-1}^{j+1} + (Sc+r)C_i^{j+1} - \frac{r}{2}C_{i+1}^{j+1} = D_i^j \quad (12)$$

Where  $A_i^j = \frac{r}{2}u_{i+1}^j + (1-r-M)u_i^j + \frac{r}{2}u_{i-1}^j + kGr\theta_i^j + kGmC_i^j$

$$B_i^j = \frac{r}{2}\theta_{i-1}^j + (P_r-r+Sk)\theta_i^j + \frac{r}{2}\theta_{i+1}^j$$

$$D_i^j = \frac{r}{2}C_{i-1}^j + (Sc-r)C_i^j + \frac{r}{2}C_{i+1}^j + SoSc r(T(i-1) - 2T(i) + T(i+1))$$

The initial and boundary conditions (9) are reduced in to finite difference form as follows:

$$\begin{aligned}
 u(i,0) &= 0; & \theta(i,0) &= 0 & \text{for all } i \\
 u(0,j) &= \cos \omega j k; & \theta(-1,j) &= \theta(1,j) + 2h \\
 u(\infty,j) &= 0; & \theta(\infty,j) &= 0
 \end{aligned}
 \tag{13}$$

Here  $r = \frac{k}{h^2}$  and  $h, k$  are mesh sizes along  $y$  and time directions respectively. Index  $i$  refers space,  $j$  for time and  $u_i^j$  is a numerical value of  $u$  at  $(i, j)$ .

With the initial and boundary conditions (13), solutions of the equations (10) and (12) have been obtained by Thomas Algorithm. In order to obtain the numerical solution with least total error and to prove the convergence of present numerical scheme, a grid independent test is applied by experimenting with different grid sizes i.e. the computation is carried out by a little varied values of  $h$ . This process is repeated until we get the results up to the desired degree of accuracy  $10^{-8}$ . No significant change is observed in the values of velocity, temperature and concentration.

#### IV. RESULTS AND DISCUSSION

Magnetic parameter  $M$  depicts the ratio of electromagnetic force to the viscous force. It is observed from the figure (1) that the velocity diminishes with an enhance in Magnetic parameter. This is due to fact that the interaction of the magnetic field with an electrically conducting fluid produces a body force known as Lorentz force, which take part in the role of a resistive type force on the velocity and this force take action in opposition to to the fluid flow when the magnetic field is applied perpendicular to it. Figure (2) confirm that an increase in  $\omega t$  leads to reduce in the velocity field. The meaning of the Soret number  $So$  is the effect of the temperature gradients to the inducing significant mass diffusion. Figures (3) and (6) demonstrate the effect of Soret number on velocity and concentration respectively. A comparative study of the graph discloses that the velocity and concentration of the fluid boosts up with

the increasing values of Soret number  $So$ . It means that Soret number accelerate the primary velocity of the flow throughout the boundary layer. Rising Soret number shows a reduce in the viscosity of the fluid. It leads to enhanced inertia effects and weakened viscous effects. Consequently the concentration and velocity of the fluid enhances. Figure (4) is drawn for various values of  $Pr$  on temperature field in the presence of heat source/sink. A comparative study of the graph shows that increasing values of Prandtl number  $Pr$ , reduces the temperature of the fluid as the higher Pandtl number fluid has comparatively lower thermal conductivity. It is a good conformity with physical fact that an enhance in  $Pr$  leads to reduce in the thermal boundary layer thickness. Further it is observed that temperature increases in the presence of heat source. But it decreases in the presence of heat sink.

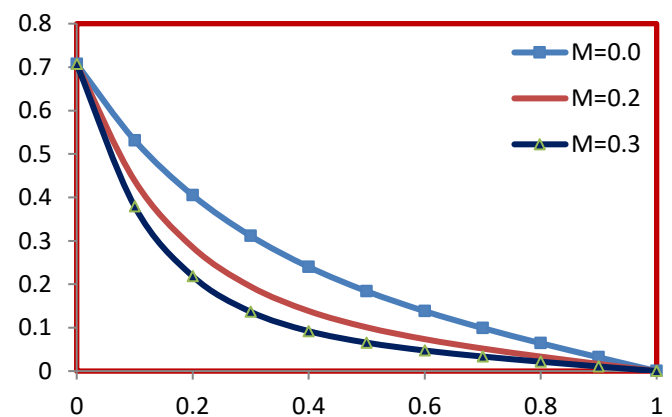


Fig 1: Effect of Magnetic parameter on velocity field  $u$  when  $Gr=5.0, Gm=5.0, Pr=0.71, Sc=0.22, S=1.0, So=1.0, wt=45$

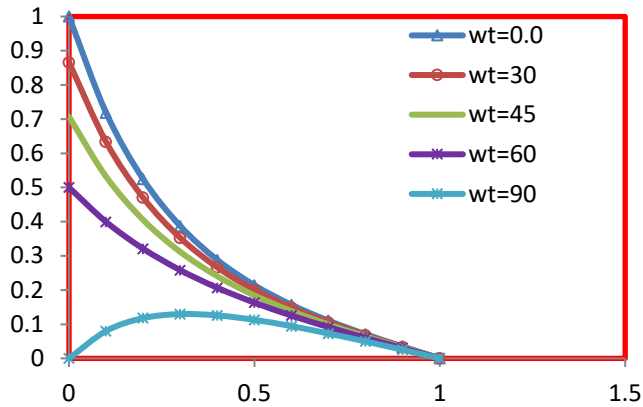


Fig 2: Effect of phase angle 'wt' on velocity field u when  $Gr=5.0$ ,  $Gm=5.0$ ,  $Pr=0.71$ ,  $Sc=0.22$ ,  $S=1.0$ ,  $So=1.0$ ,  $wt=45$

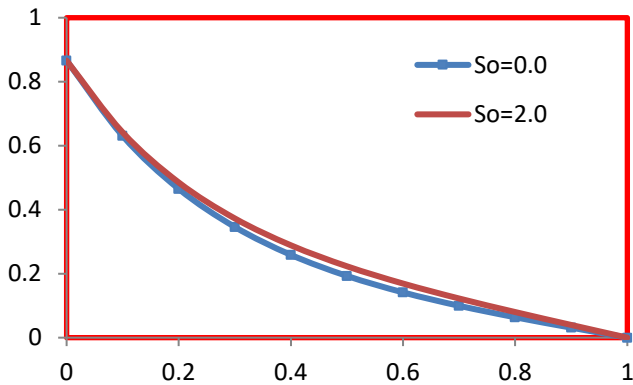


Fig 3: Effect of Soret number  $So$  on velocity field u when  $Gr=5.0$ ,  $Gm=5.0$ ,  $Pr=0.71$ ,  $Sc=0.22$ ,  $S=1.0$ ,  $So=1.0$ ,  $wt=45$

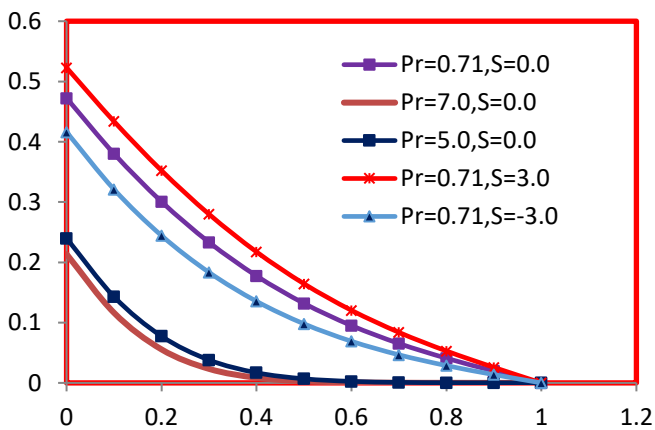


Fig 4: Effect of Prandtl number, Heat source/sink on Temperature

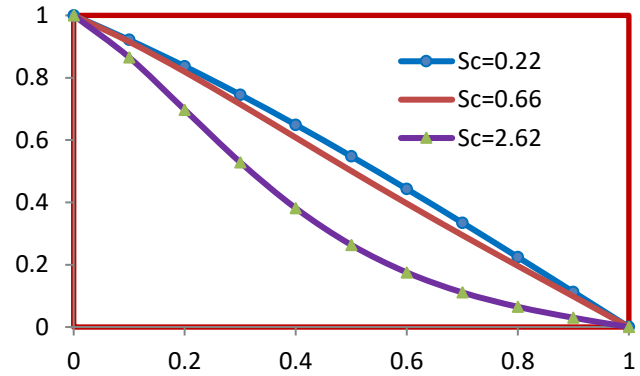


Fig 5: Effect of Schmidt number on concentration C when  $Pr=0.71$ ,  $S=1.0$ ,  $So=1.0$

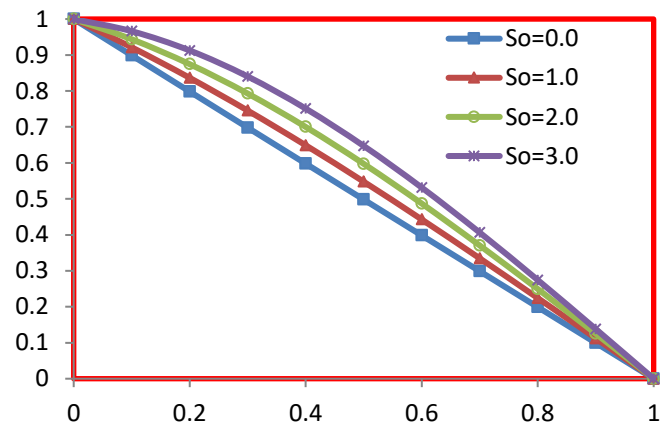


Fig 6: Effect of Soret number  $So$  on concentration C when  $Pr=0.71$ ,  $S=1.0$ ,  $Sc=1.0$

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