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Study on the Dynamic Modeling of the Turbofan Engine

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ABSTRACT

The external conditions and internal parameters of aero-engine work vary widely, and there are strict requirements of aerodynamic and strength limitation, so certain control strategies must be adopted to meet the operating requirements of the engine. Engine mathematical model is the core of engine simulation technology, the basis of analyzing engine characteristics, studying its control law, and designing and analyzing the control system. It is also an important condition for the research of integrated flight/propulsion control system. Based on the analysis of the characteristics of turbofan engine components and the thermodynamic calculation process, a component level nonlinear mathematical model of the engine with variable specific heat is established. This model can be used to calculate the steady and dynamic processes of turbofan engines under different control laws. It can also be used as the research object of multivariable control. In this paper, based on mentioning the theoretical basis for making a dynamic model of a turbofan engine, an assumption was made to create a dynamic model. Next, a geometrical structure diagram of a turbofan engine was created to create a dynamic model and the principle of the modeling method was explained. Also, based on the calculation of the characteristics of the turbofan engine and the calculation of the parts, the dynamic equation of the turbofan engine was prepared. The mathematical model of turbofan engine established in this paper has certain practicability and can be used as the basis of engine control research.

Keyword: Turbofan Engine, Mathematical Model, Part Calculation, Dynamic Model

I. INTRODUCTION

The aero-engine mathematical model is the basis for analyzing the engine characteristics, studying its control law, and designing and analyzing the control system. It is also an important condition for the semi-physical simulation of the engine control system and the integrated flight/propulsion control system, evaluating the performance of the control system and further experimental research. However, the aero-engine is a complex aero-thermodynamic system, and its internal working mechanism is quite complex. How to describe the engine with mathematical methods has always been an important subject in the research of aero-engine technology at home and abroad [1-3]. For the object to be studied, computers generally cannot directly understand and deal with it, which requires the establishment of a mathematical model that can reflect the essence of the object to be studied, and is easy to be processed by computers. The engine mathematical model is the mathematical abstraction of the engine, which describes the rules of the engine working process (steady state and dynamic) with charts, curves, formulas and differential equations. The computer processes these abstract mathematical models and outputs the data related to these models to reveal certain characteristics of the research object. This presentation can be three-dimensional (determining the level of engine awareness and CFD level of computational fluid dynamics), and because the three-dimensional display is clear and intuitive, Have been used by more and more researchers [4-6] through the analysis of the output, can be more clear understanding of the research object. Through the relationship also can be seen that the degree of precise mathematical modeling is to determine the computer simulation precision of the most key factors. From the Angle of the model, the computer simulation can be divided into three steps: the establishment of the model, model transformation and the simulation experiment of the model. The establishment of the model, you first need to according to the simulation engine to achieve the purpose of the abstract out a certain system, and to give the system boundary conditions and constraints [7]. Later, need to use a variety of disciplines of knowledge engine, the abstract out of the system with mathematical expressions to describe the content is the so-called "mathematical model", this model is the core of computer simulation. The so-called model transformation is to transform the engine mathematical expression abstracted from the previous step into a form that can be processed by the computer through various appropriate algorithms and computer languages. The content expressed in this form is the so-called "simulation model". This model is the key to computer simulation [8]. To realize this process, we can develop a new system by ourselves, and we can also use the existing simulation software on the market. Model simulation experiment, the simulation model obtained in the previous step is loaded into the computer, according to the pre-set experimental scheme to run the simulation model, get a series of simulation results, this is the so-called "model simulation experiment". For the engine, a multidisciplinary simulation system, the model can be roughly divided into several levels. First, the overall model of the engine, which regards the engine as an independent and complete system, is a basic steady-state thermodynamic model to calculate the performance and efficiency of the engine system according to the engine structure and component performance. Second, the engine system dynamics and control model. This model is a one-dimensional thermodynamic basic model with simplified structural elements, control and other discipline data. It uses component performance, geometry and dynamics data to calculate engine thrust and weight, as well as system transient characteristics [9]. Third, the engine system space and time average model, which is a two-dimensional fluid model, makes the component boundary conditions related to the entire system boundary conditions. Four, parts three-dimensional real-time accurate model, the physical process of all parts of the full three-dimensional real-time accurate simulation, this kind of model is the current aero-engine numerical simulation to pursue the goal [10]. The earliest mathematical model of aero-engine NASA Lewis developed SMOTE model in 1960s, and then GENENG and GENENGIImodel, but these three models can only calculate the steady-state performance of the engine. Later, NASA Lewis developed the three-rotor, three-bypass DYNGEN model with dynamic computing capabilities, which allowed for an arbitrary engine configuration. In the 1990s, the NPSS program of the quasi advance system numerical simulation at NASA Lewis Research Center began to adopt the object-oriented structural design and establish the general engine performance simulation program of "digital test bed", and now the MAPSS software and ONYS software have been successfully developed [11-13]. In addition, there are representatives of Russia's Central Aero Engine Research Institute and the Netherlands National Aerospace Laboratory. In particular, Dutch National Space Laboratory NAL has developed a set of engine simulation program GSP [14] based on component model under Windows environment. It adopts the flexible object-oriented programming method, and can give the structure form of the engine arbitrarily by building blocks, so as to simulate the steady state and transition state performance of the engine. In the thesis, based on the research and analysis of the literature of the predecessors in detail, the method of making a mathematical model of the turbofan engine was specifically mentioned.

II. Mathematical modelling theory of turbofan engine

2.1 The principle of creating a mathematical model and the structure of the turbofan engine

2.1.1 Structure of the turbofan engine

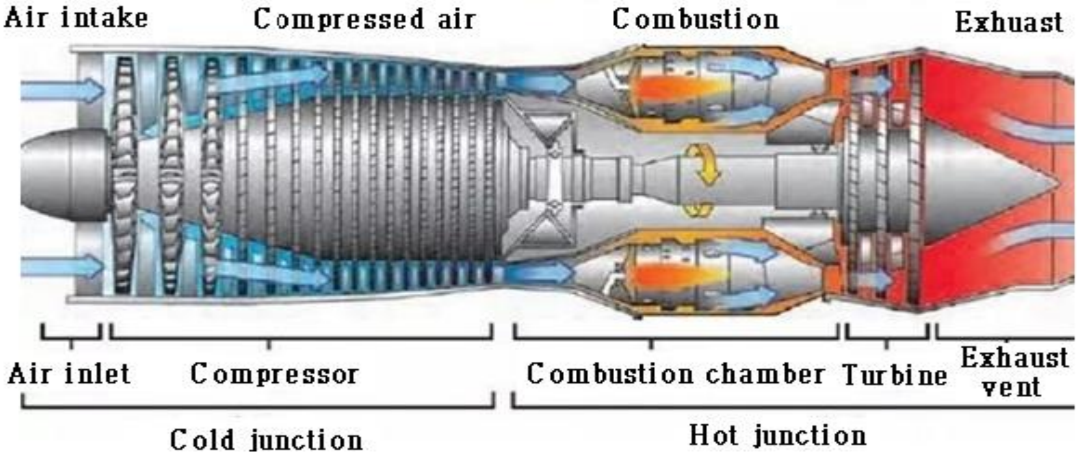

Figure 1. Turbofan engine

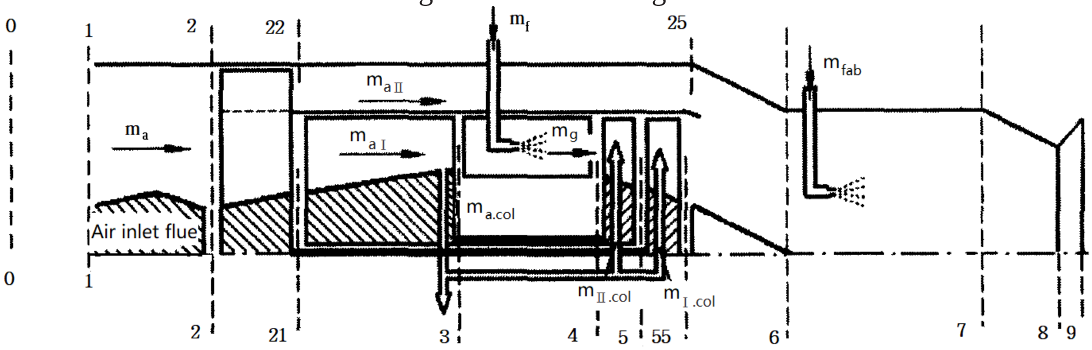

Figure 2. Schematic diagram of a turbofan engine

2.2 The principle of creating a mathematical model of the turbofan engine

The mathematical model of turbofan engine is the foundation of engine simulation. It is very important for the research of engine control law and system performance analysis. In this chapter, a component-level nonlinear mathematical model of a turbofan engine with variable specific heat and mixed exhaust is established by analytical method. According to the sequence of engine parts, the gas flow equation and thermodynamic equation are established one by one from front to back, and the thermodynamic calculation is carried out. When the parameters of the working process of components are unknown, the initial values should be given. Then, according to the constraint conditions of the engine working together, the nonlinear equations describing the aerodynamic and thermodynamic characteristics of the engine are established. Finally, by solving the nonlinear equations and finding the common working point of the engine, the parameters and performance of each section of the engine can be determined.

2.3 Mathematical model theory of the turbofan engine

Engine mathematical model is a mathematical description of the engine working process, that is, using mathematical equations, charts, function curves and other approximate reflection of the real engine, is about the engine design parameters and operating conditions and engine performance parameters of the mathematical relationship. According to different purposes and research tasks, different forms of engine mathematical models need to be derived. There are various forms of engine mathematical models, which can be divided into linear and nonlinear ones. Steady and unsteady; Deterministic and random; Continuous and divergent; Having concentrated parameters and distributed parameters; Real-time and non-real-time. According to the characteristics of the engine, the mathematical models are divided into three categories: steady state, small deviation and large deviation. The methods of establishing mathematical model of engine include analytical method and experimental method. The test method is a method to get the engine model by processing the engine test data and acquiring the engine characteristics. To establish mathematical model by analytical method requires a detailed understanding of the internal physical process of the engine and can be described by mathematical methods. In this method, a mathematical model is established by describing the differential and algebraic equations of the main components of the engine (fan, compressor, combustion chamber, turbine, nozzle, etc.) in sequence. Then, according to the relation between the parameters of various components of the engine, the balance relations (such as flow balance, pressure balance and power balance, etc.) are established. In fact, these balance relations constitute a series of nonlinear equations, which are the mathematical model of the engine.

III. Mathematical modeling of the turbofan engine

3.1 Characteristic calculation of the turbofan engine

The characteristics of engine components refer to the relationship between the main indexes of engine components and their working state, which is the basic condition of engine performance calculation and can be obtained through calculation or test.

1) Inlet characteristic

$$\sigma_i = \frac{P_2^*}{P_H^*} = \sigma(\mathrm{Ma}) \qquad (1)$$

2) Fan or Low-compressor characteristic

$m_{a.cl.cor} = f_{cl.m}(\bar{n}_{cl.cor}, \pi_{cl}^{*}, \alpha_{cl})$

$\eta_{cl} = f_{cl.\eta}(\bar{n}_{cl.cor}, \pi_{cl}^{*}, \alpha_{cl})$ (2)

Surge boundary: $\left(\pi_{cl}^{*}/q(\lambda_2)\right)_s = f(\bar{n}_{cl.cor}, \alpha_{cl})$

Fan stator blade Angle: $\alpha_{cl} = f_{cl}(\bar{n}_{cl.cor})$

3) High -compressor characteristic

$m_{a.ch.cor} = f_{ch.m}(\bar{n}_{ch.cor}, \pi_{ch}^{*}, \alpha_{ch})$

$\eta_{ch} = f_{ch.\eta}(\bar{n}_{ch.cor}, \pi_{ch}^{*}, \alpha_{ch})$ (3)

Surge boundary: $\left(\pi_{ch}^{*}/q(\lambda_{21})\right)_s = f(\bar{n}_{ch.cor}, \alpha_{ch})$

Fan stator blade Angle: $\alpha_{ch} = f_{ch}(\bar{n}_{ch.cor})$

4) Air extraction, cooling and power extraction parameters

Air intake is usually given in proportion to the air flow rate behind the high pressure compressor, that is, $m_{i.col} = K_i m_{a3}$, where K_i is a constant; The power extracted from the low pressure and high pressure rotors to the engine and aircraft accessories is $N_{l.ext}$, $N_{h.ext}$

5) Main combustor characteristic

Combustion efficiency:$\eta_b = f_{b.\eta}(far_b, P_3^{*}, T_3^{*}, T_4^{*})$ (4)

Total pressure recovery coefficient of the main combustion chamber (including flow resistance and thermal resistance):$\sigma_b = f_{b,\sigma}(\lambda_3, T_4^{*}/T_3^{*})$ (5)

Combustion chamber residual gas coefficient:$\alpha_b = (far_b \cdot L_0)^{-1}$ (6)

6) High-pressure turbine characteristics

$\Delta h_{th}^{*}/T_4^{*} = f_{th.h}(\bar{n}_{th.cor}, m_{g.th.cor})$ (7)

$\eta_{th} = f_{th.\eta}\left(\bar{n}_{th.cor}, m_{g.th.cor}\right)$ (8)

7) Low-pressure turbine characteristics

$\Delta h_{tl}^{*}/T_{45}^{*} = f_{tl.h}(\bar{n}_{tl.cor}, m_{g.tl.cor})$ (9)

$\eta_{tl} = f_{tl.\eta}\left(\bar{n}_{tl.cor}, m_{g.tl.cor}\right)$

8) Duct characteristics

Total pressure recovery coefficient of the duct:$\sigma_d = f(\lambda_d)$

9) Mixer characteristic

Total pressure recovery coefficient when air and gas are mixed: $\sigma(\lambda_m)$

10) Afterburning combustion chamber characteristics

Combustion efficiency of combustion chamber: $\eta_{ab} = f_{ab.\eta}(far_{ab}, P_6^{*}, T_6^{*}, T_7^{*})$ (10)

Total pressure recovery coefficient of afterburning combustion chamber (including flow resistance and thermal resistance):$\sigma_{ab} = f_{ab.\sigma}(\lambda_6, T_7^{*}/T_6^{*})$ (11)

11) Nozzle characteristics

Total pressure recovery coefficient of nozzle is σ_e.

12) Feature sizes

The area of each characteristic section A_2, A_{21}, A_{22}, A_{25}, A_{55}, A_6, and the volume V_b, V_{II}, V_m, V_{ab} of the main combustion chamber, outer bypass, mixer and afterburner.

13) Physical parameters

The moment of inertia of low and high pressure rotors J_l, J_h.

3.2 Mathematical modelling of the turbofan engine

3.2.1 Basic assumption

The thermodynamic process of aeroengine is complex. In order to simplify the derivation of the mathematical model of the engine, the following assumptions are made during the modeling process:

① The gas parameters of the engine section only change along the axial direction and have no change in the radial direction;

② The viscosity (the influence of Reynolds number on compressor and turbine characteristics is ignored) and the mass force are not considered in all the aerodynamic equations;

③The aerothermodynamic process in blade machinery and heat exchange is regarded as steady;

④ Ignoring combustion delay.

The gas flow equation and thermodynamic equation are established one by one according to the sequence of engine components. Finally, the mathematical model of turbofan engine with mixed exhaust is established through the common working equation.

3.2.2 Mathematical modelling

3.2.2.1 Parts calculation

1) Inlet calculation

Inlet calculation refers to the calculation of airflow parameters at the inlet outlet with known flight altitude H and Mach number Ma. Static pressure and static temperature under standard atmospheric conditions can be according to international standard atmospheric conditions To=T(H) and Po = P (H).

The total enthalpy of air inlet $h_1^*$is:

$$h_1^* = h_0 + c_0^2/2 \qquad (12)$$

From $T_1^* = \varphi_T(h_1^*)$, calculate the inlet temperature T_1^*, and then calculate $\phi(T_1^*)$ from T_1^*.

The total inlet pressure is calculated according to isentropic process.

$$P_1^* = P_0 e^{[\phi(T_1^*) - \phi(T_0)]/R_0} \qquad (13)$$

Then, the total pressure P_2^* at the inlet outlet and the exit entropy S_2^* are calculated by the total pressure recovery coefficient. Then, the exit parameters are:

$$T_2^* = T_1^*$$
$$P_2^* = \sigma_i P_1^*$$
$$h_2^* = h_1^*$$
$$S_2^* = \phi(T_2^*) - R_0 \ln(P_2^*/P_0) \qquad (14)$$

2) Low pressure compressor calculation

Select fan pressure ratio π_{cl}^* and relative conversion speed $\bar{n}_{cl.cor}$, the fan conversion flow rate $m'_{a.cl.cor}$ and efficiency η'_{cl}can be obtained by binary interpolation based on fan characteristics, and the flow rate $m_{a.cl}$ and efficiency η_{cl}can be obtained after correction. If the fan inlet parametersT_2^*, P_2^*, h_2^*, s_2^* and π_{cl}^* and η_{cl} are known, then the calculation method of compressor outlet parameters can be obtained following equation.

$$P_{21}^* = \pi_{cl}^* P_2^*$$
$$h_{21}^* = h_2^* + (h_{21.p}^* - h_2^*)/\eta_{cl}$$
$$T_{21}^* = \varphi_T(h_{21}^*)$$
$$S_{21}^* = \phi(T_{21}^*) - R\ln(P_{21}^*/P_0) \qquad (15)$$

Where: $h_{21.p}^*$ is the total enthalpy at the exit under isentropic compression.

3) High pressure compressor calculation

Select fan pressure ratio π_{ch}^* and relative conversion speed $\bar{n}_{ch.cor}$, the fan conversion flow rate $m'_{a.ch.cor}$ and efficiency η'_{ch}can be obtained by binary interpolation based on fan characteristics, and the flow rate $m_{a.ch}$ and efficiency η_{ch}can be obtained after correction. If the fan inlet parametersT_{21}^*, P_{21}^*, h_{21}^*, s_{21}^* and π_{ch}^* and η_{ch} are known, then the calculation method of compressor outlet parameters can be obtained following equation.

$$P_3^* = \pi_{ch}^* P_{21}^*$$
$$h_3^* = h_{21}^* + (h_{3.p}^* - h_{21}^*)/\eta_{ch}$$
$$T_3^* = \varphi_T(h_3^*)$$

$$S_3^* = \phi(T_3^*) - R\ln(P_3^*/P_0) \quad (16)$$

Where: $h_{3.p}^*$ is the total enthalpy at the exit under isentropic compression and can calculate the airflow velocity Ma3 or λ_3.

4) Main combustion chamber calculation

The energy balance equation of the main combustion chamber is:

$$m_{a3}h_3^* + m_f Hu_4 \eta_b = (m_{a3} + m_f)h_4^* \quad (17)$$

Oil and gas rate is:

$$far_b = m_f/m_{a3} \quad (18)$$

According to the energy balance equation, the gas-gas ratio equation of combustion chamber can be obtained:

$$far_b = [h_4^*(T_4^*, far_b) - h_3^*]/[Hu_4(T_4^*)\eta_b - h_4^*(T_4^*, far_b)]$$
$$m_f = m_{a,3} far_b$$
$$P_4^* = \sigma_b P_3^* \quad (19)$$

Total enthalpy at the combustion chamber exit:

$$h_4^*(T_4^*, far_b) = (h_{a4}^* + far_b \cdot h_{e4}^*)/(1 + far_b) \quad (20)$$

Where: h_{a4}^* is the air enthalpy, h_{e4}^* is the equivalent enthalpy of fuel oil.

Exit entropy function:

$$\phi_4(T_4^*, far_b) = [\phi_{a4}(T_4^*) + far_b \cdot \phi_{e4}(T_4^*)]/(1 + far_b) \quad (21)$$

Export entropy:

$$S_4^* = \phi_4(T_4^*) - R_4\ln(P_4^*/P_0) \quad (22)$$

The gas flow at the combustion chamber outlet is:

$$m_{g4} = m_{a3} + m_f \quad (23)$$

5) High pressure turbine calculation (set flow rate)

The calculation of high-pressure turbines is divided into three parts: component calculation, mixing calculation of main flow and cooling air, and calculation of volumetric effect (dynamic process is taken into account). In order to take advantage of the characteristics of high-pressure turbines, the inlet of high-pressure turbines is selected to convert the initial flow value $m_{g4.cor}$. Turbine components, mixer and volume effect were calculated successively, in which the cooling air flow rate was $m_{a.ch.col.th}$, and the exit flow parameter $far_5, T_5^*, P_5^*, h_5^*, s_5^*$was obtained.

6) Low-pressure turbine calculation (assumed flow rate)

In order to take advantage of the characteristics of low-pressure turbines, the inlet $m_{g5.cor}$ of low-pressure turbines is selected as the initial value of converted flow, and the calculation method is the same as that of high-pressure turbines. The cooling air flow is $m_{a.ch.col.tl}$, and the exit air flow parameters $far_{55}, T_{55}^*, P_{55}^*, h_{55}^*, s_{55}^*$ are obtained.

7) Outer bypass calculation

Outer bypass inlet flow: $m_{a22} = m_{a.f} - m_{a.c}$ (24)

According to the absolute energy flow, the total pressure at the outlet of the outer bypass is $P_{25}^* = \sigma_d P_{22}^*$, the total temperature at the outlet of the outer bypass $h_{25}^* = h_{22}^*$. According to the relationship between entropy function and temperature, the outlet entropy is following:

$$S_{25}^* = \phi_{25}(T_{25}^*) - R_{25}\ln{(P_{25}^*/P_0)} \quad (25)$$

8) Mixing chamber calculation

(1) Design point calculation

The Mach number Ma_{55} of the inner bypass is specified at the design point, which is used to determine the inner and outer bypass outlet areas A_{25} and A_{55}. The outlet area of the mixing chamber is $A_6 = A_{25} + A_{55}$, which is taken as the area calculated at non-design points. The design point requires that the static pressure at the outlet of the inner and outer bypass be equal, that is, $P_{25} = P_{55}$.

(2) Outlet parameters of mixing chamber at non-design points

Partial exit parameters of mixing chamber (adiabatic process):

$$m_{g6} = m_{g55} + m_{g25}$$
$$h_6^* = (m_{a25}h_{25}^* + m_{g55}h_{55}^*)/m_{g6}$$
$$far_6 = m_f/(m_{a25} + m_{a55})$$
$$T_6^* = \varphi_T(h_6^*, far_6) \quad (26)$$

Where, the outlet air flow of the inner bypass is:

$$m_{a55} = m_{g55}/(far_{55} + 1)$$

According to the impulse formula, the outlet flow parameter is:

$$P_6 = m_{g.6}R_6T_6/(C_{a.6}Ma \cdot A_6)$$
$$P_6^* = P_6(1 + \frac{k-1}{2}Ma^2)^{k/(k-1)}$$
$$s_6^* = s(P_{6,}^*far, h_6^*) \quad (27)$$

9) Afterburner calculation

(1) No afterburner calculation

When the engine does not turn on afterburner, there is only flow resistance loss, then the outlet pressure of afterburner is $P_7^* = \sigma_{ab}P_6^*$; The total temperature at the outlet of the afterburner and the ratio of oil to gas are $T_7^* = T_6^*$, $far_7 = far_6$, respectively. The entropy at the outlet of the afterburner is following:

$$S_7^* = \phi_7(T_7^*, far_7) - R_7\ln{(P_7^*/P_0)} \quad (28)$$

(2) Afterburner calculation

The characteristics of afterburner are $\eta_{ab} = far_{ab}, P_6^*, T_6^*, T_7^*$ and $\sigma_{ab} = f(\lambda_6, T_7^*/T_6^*)$.

Energy balance equation of afterburner:

$$m_{f.ab}Hu_7\eta_{ab} + m_{g6}h_6^* = (m_{g6} + m_{fab})h_7^* \quad (29)$$

Where far_{ab} is the oil-gas ratio of afterburner.

$far_{ab} = m_{fab}/m_{a6}$

m_{a6} is the inlet air flow of afterburner.

$m_{a6} = m_{g6} - m_{g5}far_5/(1 + far_5)$

Outlet pressure of afterburner is:

$$P_7^* = \sigma_{ab}P_6^* \qquad (30)$$

Outlet flow of afterburner is:

$$m_{a7} = m_{g6} + m_{fab} \qquad (31)$$

Oil-gas ratio at afterburner outlet is:

$$far_7 = (m_f + m_{fab})/m_{acl} \qquad (32)$$

10) Receiving and expanding nozzle calculation

The flow state of the retraction-expansion nozzle, also known as the Laval nozzle, includes subcritical flow, critical flow and supercritical flow. The supercritical flow includes complete expansion, underexpansion and overexpansion. The flow state must be judged first, and then the corresponding calculation method should be selected according to the flow state. According to the nozzle is an absolute energy process, the relevant airflow parameters are as follows:

$$h_8^* = h_9^* = h_7^*$$
$$T_8^* = T_9^* = T_7^*$$
$$m_{g8} = m_{g9} = m_{g7}$$
$$far_8 = far_9 = far_7 \qquad (33)$$

(1) Design point calculation

Throat area is:

$$A_8 = m_{g8}/(\rho_8 C_8) \qquad (34)$$

Where $\rho_8 = P_7^*(T_8/T_7)^{k_8/(k_8-1)}/(R_8T_8)$

When the nozzle reaches complete expansion, the static pressure of the outlet section is equal to the local atmospheric pressure, that is, $P_9 = P_a$.According to the isentropic process, T_9, h_9, and a_9 are calculated iteratively, then the nozzle outlet area is obtained:

$$A_9 = A_8/q(Ma_9) \qquad (35)$$

Where $Ma_9 = c_9/\sqrt{K_9R_9T_9}$, $c_9 = \sqrt{2(h_7^* - h_9)}$

(2) Non-design point calculation

When the nozzle throat critical flow, $Ma_9 = 1$. Iteratively solve the critical throat static temperature T_8 and a_8, then the throat static pressure P_8 is:

$$P_8 = m_{g7}R_8T_8/(A_8a_8) \qquad (36)$$

The total pressure $P_7'^*$ at nozzle inlet is calculated according to isentropic process.

$$P_7'^* = P_8(T_7/T_8^*)^{k_8/(k_8-1)} \qquad (37)$$

The flow state of the nozzle can be judged by comparing the static pressure P_8 at the critical point of the nozzle throat with the atmospheric back pressure P_a.

3.2.2.2 Balance equation of the turbofan engine

According to the calculation of each component, in order to carry out the thermodynamic calculation smoothly, the initial values of six parameters, namely n_{cl}, n_{ch}, π_{cl}^*, π_{ch}^*, $m_{g.th}$, $m_{g.tl}$were selected for the engine.

According to the common working relationship of engine components, the turbofan engine should follow the conditions of power balance, flow balance and pressure balance, and the correctness of the six initial values can be checked according to the balance conditions. For this reason, the following six equilibrium equations are used in the mathematical model.

1) High-pressure shaft power balance equation

$$N_{th} - N_{ch} - N_{h.ext} - \left(\frac{\pi}{30}\right)^2 J_h n_{ch} \frac{dn_{ch}}{dt} = 0 \qquad (38)$$

Where $N_{th} = m_{g4}(h_4^* - h_5^*)$, $N_{ch} = m_{a.ch}(h_3^* - h_{21}^*)$

2) Balance equation of gas flow at the outlet of combustor and inlet of high-pressure turbine

$$m_{g4} - m_{g.th} = 0 \qquad (39)$$

Where $m_{g4} = m_{a.ch} + m_f - m_{ch.col}$

3) Low pressure shaft power balance equation

$$N_{tl} - N_{cl} - N_{l.ext} - \left(\frac{\pi}{30}\right)^2 J_l n_{cl} \frac{dn_{cl}}{dt} = 0 \qquad (40)$$

Where $N_{tl} = m_{g5}(h_5^* - h_{55}^*)$, $N_{cl} = m_{a.cl}(h_{21}^* - h_2^*)$

4) Balance equation of high-pressure turbine outlet gas flow and low-pressure turbine inlet gas flow

$$m_{g5} - m_{g.tl} = 0 \qquad (41)$$

Where $m_{g5} = m_{g4} + m_{ch.col}$

5) The static pressure balance equation of the inner and outer bypass at the inlet of the mixing chamber

$$P_{25} - P_{55} = 0 \qquad (42)$$

6) Balance equation of total pressure at the outlet of the afterburner and the total pressure at the nozzle inlet

$$P_7^* - P_7'^* = 0 \qquad (43)$$

According to the principle of the engine, the six balance equations of the engine can uniquely determine the working state of the engine, that is, the engine parameters can be expressed by six independent parameters. Note that X $= [x_1, x_2, \ldots, x_6]^T$ is the column vector of the six independent parameters of the engine, and the balance equation is expressed as:

$$\begin{cases} f_1(x_1, x_2, \ldots, x_6) = 0 \\ f_2(x_1, x_2, \ldots, x_6) = 0 \\ \qquad \cdots \\ f_6(x_1, x_2, \ldots, x_6) = 0 \end{cases} \qquad (44)$$

As long as the balanced nonlinear equations are solved, the parameters of each section of the engine can be obtained. The choice of six independent parameters usually has the following types:

When T_4 or m_f is given, the independent parameters are:

$n_{cl}, n_{ch}, \pi_{cl}^*, \pi_{ch}^*, m_{g.th}, m_{g.tl}$

When n_{ch} or n_{cl} is given, the independent parameters are:

$T_4^*, n_{cl}\ (n_{ch}), \pi_{cl}^*, \pi_{ch}^*, m_{g.th}, m_{g.tl}$

For turbine characteristics in the form of pressure ratio, π_{th}^*, π_{tl}^* can be selected as an independent parameter.

3.2.2.3 Calculation of engine performance parameters

When the airflow parameters of each section of the engine are obtained, the thrust and fuel consumption rate of the engine can be calculated.

Total thrust is following;

$$F_G = m_{g9} V_9 + (P_9 - P_a) A_9 \qquad (45)$$

Net thrust is following;

$$F_N = m_{g9}V_9 + (P_9 - P_a)A_9 - m_{a.f}V_0 \qquad (46)$$

Fuel consumption rate (per unit fuel consumption rate) is following:

$$\text{sfc} = \frac{3600(m_f + m_{f.ab})}{F_N} \qquad (47)$$

Bypass ratio

$$\text{B} = \frac{m_{a22}}{m_{a.c}} \qquad (48)$$

IV. DISCUSSION

The turbofan engine mathematical model is the basis for analyzing the engine characteristics, studying its control law, and designing and analyzing the control system. It is also an important condition for the semi-physical simulation of the engine control system and the integrated flight/propulsion control system, evaluating the performance of the control system and further experimental research. Therefore, in this paper, based on mentioning the theoretical basis for making a dynamic model of a turbofan engine, an assumption was made to create a dynamic model. Next, a geometrical structure diagram of a turbofan engine was created to create a dynamic model and the principle of the modeling method was explained. Also, based on the calculation of the characteristics of the turbofan engine and the calculation of the parts, the dynamic equation of the turbofan engine was prepared.

V. CONCLUSION

The external conditions and internal parameters of aero-engine work vary widely, and there are strict requirements of aerodynamic and strength limitation, so certain control strategies must be adopted to meet the operating requirements of the engine. Engine mathematical model is the core of engine simulation technology, the basis of analyzing engine characteristics, studying its control law, and designing and analyzing the control system. It is also an important condition for the research of integrated flight/propulsion control system. Based on the analysis of the characteristics of turbofan engine components and the thermodynamic calculation process, a component level nonlinear mathematical model of the engine with variable specific heat is established. This model can be used to calculate the steady and dynamic processes of turbofan engines under different control laws. It can also be used as the research object of multivariable control. The mathematical model of turbofan engine established in this paper has certain practicability and can be used as the basis of engine control research.

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