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Study on the Dynamic Modelling of the Rotor System of the Turbofan Engine with a Double Rotor Structure

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ABSTRACT

The gas turbine engine is directly related to whether the engine, as the heart of the plane, can work normally and the flight stability of the plane. In addition, the rotor system is also a core part of the gas turbine engine and is a fundamental vibration source. Therefore, in this dissertation, the task of research was to create a mathematical model of the rotor system of the turbofan engine with a double rotor structure. First, the model of a single rotor system was made. The modeling method of a simple single rotor system with one disk and two supports was specifically mentioned. Next, the mathematical modelling method of the rotor system of the turbofan engine with a double rotor structure was mentioned. The gas turbine engine rotor system is composed of a low-pressure rotor part and a high-pressure rotor part. Firstly, the mathematical model of low pressure rotor system of double rotor system was built. In modelling, the effects of asymmetry, mass eccentricity, and gyroscopic effect of the low-pressure rotor system were fully considered and the differential equation of the low-pressure rotor system was written by utilizing Lagrange equation. Next, a differential equation of a high pressure rotor system was formulated, considering sufficient conditions such as low pressure rotor system.

Keywords : Double Rotor System, Eccentricity, High Pressure Turbine, Low Pressure Turbine, Mathematical Model

I. INTRODUCTION

Gas turbine -engine is described as the "heart" of aircraft, its performance directly affects the safety and reliability of aircraft flight, and the rotor system is the core of gas turbine -engine. Once the rotor equipment malfunction, the accident is huge, even catastrophic. In order to improve its thrust-to-weight ratio, modern gas turbine -engine constantly reduce the weight of engine parts and increase their load. Gas turbine -engine is a typical rotor as the main body of high-speed rotating machinery, in the work, the rotor system often vibration, its harm mainly produces noise, accelerate wear, reduce service life, serious vibration will lead to the fracture of the rotor, resulting in accidents. According to statistics, 80% of

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the engine vibration failure is caused by the failure of the rotating part, the causes and forms of vibration caused by different types of engines are not the same. The practice shows that the engine vibration fault caused by rotor-support system has become an important key technology in troubleshooting. Since then, the predecessors have already conducted research projects on modeling the rotor system of a high-speed rotating machine and analyzing the dynamics characteristics. Since Jeffcott is the first to conduct a research project on the vibration of the rotor system, based on linear system theory of rotor dynamics obtained the very big development, involving the main problems include the determination of critical speed, unbalance response calculation, movement stability, parameter identification, the rotor balance and so on. Unbalance of rotor system is one of the main causes of vibration. In the manufacturing process of the engine, due to uneven material, machining and installation errors, there is inevitably eccentricity, that is, the center of mass deviates from the nominal center of the rotor. Unbalance of rotor system is one of the main causes of vibration. In the manufacturing process of the engine, due to uneven material, machining and installation errors, there is inevitably eccentricity, that is, the center of mass deviates from the nominal center of the rotor. Sudhakara[1] used the finite element method to model the system and studied the influence of different types of oil film supports on the complex eigenvalues of the rotor bearing system. Fawzi[2] applied the finite element method to study the finite element modeling method of the rotor-bearing system. Dutt[3] used the finite element method to create a model of the rotor bearing support system under the condition that the mass of the support was considered and the mass of the rotor was ignored. Kang[4] created a three-dimensional finite element model of the rotor, bearing, and support, respectively, and studied the relationship between the support type and the stability of the rotor system. Nelson[5] obtained the finite element memory and mass matrix of the Rayleigh beam-shaft model by calculating the gyroscopic effect and rotational inertia of the rotating shaft. Based on this, the finite element formula of the Timoshenko beam-shaft model was derived [6]. Oncescu[7] studied and analyzed the stability of an asymmetric rotor-bearing system by combining the finite element method and the time transfer function method based on the Floquet theory. Mohiuddin[8] studied the stability problem of the rotor-bearing system by using the mode reduction method by applying the finite element method and the Lagrange method at the same time. Friswell[9] used the finite element method to model the rotor system and calculated the critical speed of the rotor bearing system in consideration of the bearing dynamic characteristic coefficient according to the change of rotational speed. D.Mku[10] used Timoshenko beam elements to determine the critical speed and stability of the rotor-bearing system in various states. Madhumita[11] calculated the critical speed of the rotor system through the cambell diagram and studied the stability of the damped rotor bearing system based on the finite element method. Pingchao Yu[12] studied the dynamic responses of the complicated dual-rotor systems at instantaneous and windmilling statuses when FBO event occurs. A simplified dynamic model of a dual-rotor system coupled with blade disk is built by Zhenyong Lu[13]. In this paper, a mathematical model of the rotor system of the turbofan engine with a double rotor structure was created based on the research and analysis of the precedence literatures in detail.

II. Dynamic Modelling of Single Rotor System

2.1 Dynamic model of single rotor system

The rotor system consists of a turntable and two supports, and a single disc with eccentric mass is placed in the center of the shaft.

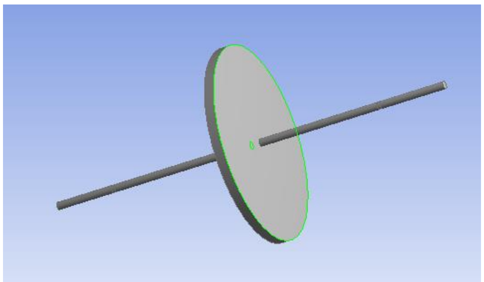

Figure 1. 3D single rotor model

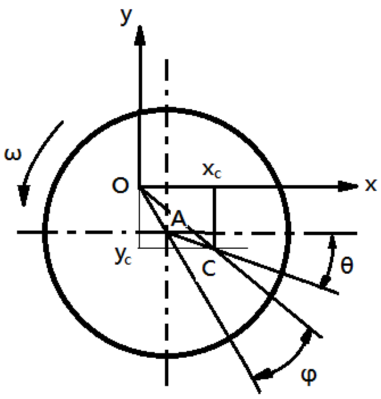

Figure 2. Rotor eccentric model

2.2 Dynamic equation of single rotor system

In the figure 2, C is the center of mass of the turntable, A is the center of the turntable, xc, yc are the coordinates of the center of mass of the turntable, CA is the mass eccentricity (CA=Le), and ω is the rotation angular velocity. The relationship of the centroid coordinates of the turntable (x_c, y_c) and the fixed coordinates(x, y) is;

$$\begin{cases} x_c = x + L_e\cos\theta \\ y_c = y + L_e\sin\theta \end{cases} \qquad (1)$$

Where $\theta = \omega t$

When the elastic force of the rotation axis is F, the motion equation of the center of mass is denoted as.

$$\begin{cases} m\ddot{x}_c = -kx \\ m\ddot{y}_c = -ky \end{cases} \qquad (2)$$

Where

$$\begin{cases} \ddot{x}_c = \ddot{x} - L_e\omega^2 \sin\omega t \\ \ddot{y}_c = \ddot{y} - L_e\omega^2 \cos\omega t \end{cases} \qquad (3)$$

By substituting equation (3) into equation (2), the motion differential equation of axial center C is obtained.

$$\begin{cases} \ddot{x} + \frac{k}{m}x = L_e\omega^2 \sin\omega t \\ \ddot{y} + \frac{k}{m}y = L_e\omega^2 \cos\omega t \end{cases} \qquad (4)$$

If $\omega_n = \sqrt{\frac{k}{m}}$, the formula (4) can be written as follows.

$$\begin{cases} \ddot{x} + \omega_n^2 x = L_e\omega^2 \sin\omega t \\ \ddot{y} + \omega_n^2 y = L_e\omega^2 \cos\omega t \end{cases} \qquad (5)$$

Equation (5) is the forced motion equation of the mass eccentric disk, and the right side of the equation is equivalent to the exciting force generated by the eccentric mass, i.e. the unbalanced mass.

In the form of complex variables, equation (5) can be written as:

$$\ddot{z} + \omega_n z = L_e\omega^2 e^{i\omega t} \qquad (6)$$

The solution of equation (6),

$$z = Ae^{i\omega t} \qquad (7)$$

Substituting Equation (7) into Equation (6), the amplitude can be obtained as;

$$|A| = \left|\frac{L_e\omega^2}{\omega_n^2 - \omega^2}\right| = \left|\frac{L_e(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2}\right| \qquad (8)$$

Then the response of the mass eccentric disk to the unbalanced mass is:

$$z = \frac{L_e(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2} e^{i\omega t} \qquad (9)$$

III. Dynamic modelling of dual rotor system

3.1 Dynamic model of the dual rotor system

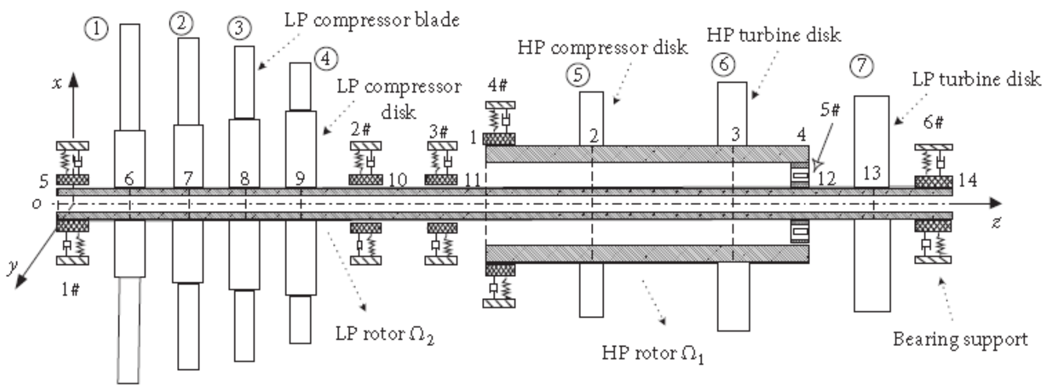

Figure 3. Dual rotor system model

In the actual double-rotor structure, the mechanical model of the rotor system is simplified according to the characteristics of the asymmetrical distribution of the turntable and the different support modes of the high-low pressure rotors. In the process of simplification, the influence of high and low pressure rotor support asymmetry, mass eccentricity and gyro effect are considered. In the model, each support is simplified as a spring damping system.

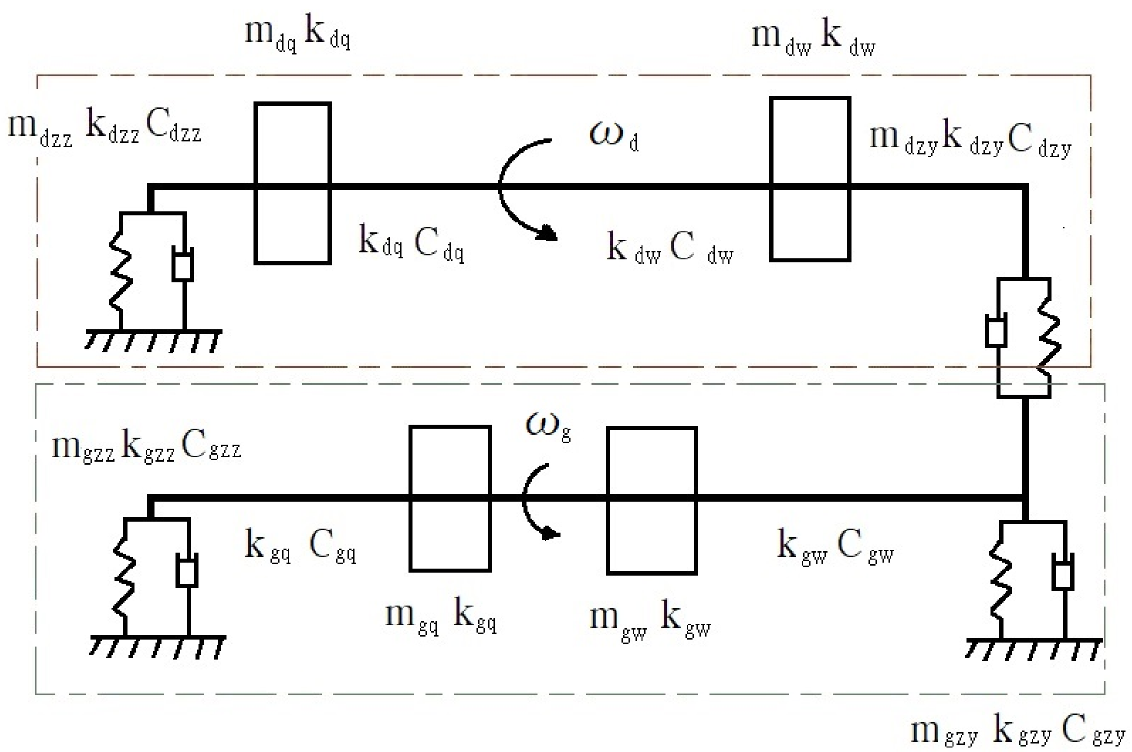

Figure 4. Mechanical model of dual rotor system

3.2 Dynamic equation of dual rotor system

3.2.1 Dynamic modelling of a low-pressure rotor system

~ Parameters definition

In figure 4, $m_{dzz}, m_{dzy}, m_{dq}, m_{dw}$ are the equivalent mass of the low-pressure rotor support, the low-pressure compressor, the low-pressure turbine disc and the low-pressure rotor support.

$k_{dzz}, k_{dzy}, k_{dq}, k_{dw}$ are the equivalent stiffness of low pressure rotor support, low pressure compressor, low pressure turbine disc and low pressure rotor support.

$C_{dzz}, C_{dzy}, C_{dq}, C_{dw}$ are equivalent damping of low pressure rotor support, low pressure compressor, low pressure turbine disc and low pressure rotor support.

$J_{dzz}, J_{dzy}, J_{dq}, J_{dw}$ are the diameter moment of inertia and polar moment of inertia of the low-pressure compressor and low-pressure turbine disk.

μ_{dq}, μ_{dw} are the eccentricity of low-pressure compressor and low-pressure turbine disc.

ω_d, l_d are the speed and shaft length of the low-pressure rotor.

$P_{dqx}, P_{dqy}, P_{dwx}, P_{dwy}$ are the external forces in X and Y directions of the low pressure compressor and the low pressure turbine disc.

F_{irmx}, F_{irmy} are the reaction forces of the intermediate bearing.

~ Dynamic modelling of a low-pressure rotor system

The kinetic energy, potential energy and dissipation functions of low-pressure compressor, low-pressure turbine disc and bearing in the Figure 4 are:

$$T_{dq} = \frac{1}{2} m_{dq} \left[\left(\dot{x}_{dq} - u_{dq}\omega_d \sin \omega_d t \right)^2 + \left(\dot{y}_{dq} - u_{dq}\omega_d \cos \omega_d t \right) \right] + \frac{1}{2} \left[J_{dqd} \left(\dot{\theta}_{dx}^2 + \dot{\theta}_{dy}^2 \right) + J_{dqd}\omega^2 - 2J_{dqp}\omega \dot{\theta}_{dy} \theta_{dx} \right] \quad (10)$$

$$T_{dw} = \frac{1}{2} m_{dw} [(\dot{x}_{dw} - u_{dw}\omega_d \sin \omega_d t)^2 + (\dot{y}_{dw} - u_{dw}\omega_d \cos \omega_d t)] + \frac{1}{2} \left[J_{dwd} \left(\dot{\theta}_{dx}^2 + \dot{\theta}_{dy}^2 \right) + J_{dwd}\omega^2 - 2J_{dwp}\omega \dot{\theta}_{dy} \theta_{dx} \right] \quad (11)$$

$$T_{dz} = \frac{1}{2} m_{dzz} \left(\dot{x}_{dzz}^2 + \dot{y}_{dzz}^2 \right) + \frac{1}{2} m_{dzy} \left(\dot{x}_{dzy}^2 + \dot{y}_{dzy}^2 \right) \quad (12)$$

$$U_{dq} = \frac{1}{2} k_{dq} \left\{ \left(x_{dq} - (x_{dzz} + x_{dzy})/2 \right)^2 + \left(y_{dq} - (y_{dzz} + y_{dzy})/2 \right)^2 \right\} \quad (13)$$

$$U_{dw} = \frac{1}{2} k_{dw} \left\{ \left(x_{dw} - (x_{dzz} + x_{dzy})/2 \right)^2 + \left(y_{dw} - (y_{dzz} + y_{dzy})/2 \right)^2 \right\} \quad (14)$$

$$U_{dz} = \frac{1}{2} k_{dzz} \left(x_{dzz}^2 + y_{dzz}^2 \right) + \frac{1}{2} k_{dzy} \left(x_{dzy}^2 + y_{dzy}^2 \right) \quad (15)$$

$$D_{dq} = \frac{1}{2} c_{dq} \left\{ \left[\dot{x}_{dq} - (\dot{x}_{dzz} + \dot{x}_{dzy})/2 \right]^2 + \left[\dot{y}_{dq} - (\dot{y}_{dzz} + \dot{y}_{dzy})/2 \right]^2 \right\} \quad (16)$$

$$D_{dw} = \frac{1}{2} c_{dw} \left\{ \left[\dot{x}_{dw} - (\dot{x}_{dzz} + \dot{x}_{dzy})/2 \right]^2 + \left[\dot{y}_{dw} - (\dot{y}_{dzz} + \dot{y}_{dzy})/2 \right]^2 \right\} \quad (17)$$

$$D_{dz} = \frac{1}{2} c_{dzz} \left(\dot{x}_{dzz}^2 + \dot{y}_{dzz}^2 \right) + \frac{1}{2} c_{dzy} (\dot{x}_{dzy}^2 + \dot{y}_{dzz}^2) \quad (18)$$

Where

$$\theta_{dx} = \frac{y_{dzz} - y_{dzy}}{l_d}, \theta_{dy} = \frac{x_{dzz} - x_{dzy}}{l_d} \quad (19)$$

According to the Lagrange equation of non-conservative systems:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = Q_j \quad (20)$$

The differential equation of the low-pressure rotor system is obtained as follows:

$$m_{dq}\ddot{x}_{dq} + k_{dq}\left[x_{dq} - \left(x_{dzz} + x_{dzy}\right)/2\right] + C_{dq}\left[\dot{x}_{dq} - \left(\dot{x}_{dzz} + \dot{x}_{dzy}\right)/2\right] = m_{dq}u_{dq}\omega_d^2\cos\omega t + P_{dqx} \quad (21)$$

$$m_{dq}\ddot{y}_{dq} + k_{dq}\left[y_{dq} - \left(y_{dzz} + y_{dzy}\right)/2\right] + C_{dq}\left[\dot{y}_{dq} - \left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)/2\right] = m_{dq}u_{dq}\omega_d^2\sin\omega t + P_{dqy} - m_{dq}\,g \quad (22)$$

$$m_{dw}\ddot{x}_{dw} + k_{dw}\left[x_{dw} - \left(x_{dzz} + x_{dzy}\right)/2\right] + C_{dw}\left[\dot{x}_{dw} - \left(\dot{x}_{dzz} + \dot{x}_{dzy}\right)/2\right] = m_{dw}u_{dw}\omega_d^2\cos\omega t + P_{dwx} \quad (23)$$

$$m_{dw}\ddot{y}_{dw} + k_{dw}\left[y_{dw} - \left(y_{dzz} + y_{dzy}\right)/2\right] + C_{dw}\left[\dot{y}_{dq} - \left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)/2\right] = m_{dw}u_{dw}\omega_d^2\sin\omega t + P_{dwy} - m_{dw}g$$

(24)

$$m_{dzz}\ddot{x}_{dzz} + k_{dzz}x_{dzz} + C_{dzz}\dot{x}_{dzz} + J_{dq}\frac{\ddot{x}_{dzy} - \ddot{x}_{dzz}}{l_d^2} + J_{dw}\frac{\ddot{x}_{dzy} - \ddot{x}_{dzz}}{l_d^2} - J_{dq}\omega_d\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} - J_{dw}\omega_d\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} - \frac{1}{2}k_{dq}\left[x_{dq} - \frac{\left(x_{dzz} + x_{dzy}\right)}{2}\right] - \frac{1}{2}k_{dw}\left[x_{dw} - \frac{\left(x_{dzz} + x_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dq}\left[\dot{y}_{dq} - \frac{\left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dw}\left[\dot{y}_{dw} - \frac{\left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)}{2}\right] = 0$$

(25)

$$m_{dzz}\ddot{y}_{dzz} + k_{dzz}y_{dzz} + C_{dzz}\dot{y}_{dzz} + J_{dq}\frac{\ddot{y}_{dzy} - \ddot{y}_{dzz}}{l_d^2} + J_{dw}\frac{\ddot{y}_{dzy} - \ddot{y}_{dzz}}{l_d^2} - J_{dq}\omega\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} - J_{dw}\omega\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} - \frac{1}{2}k_{dq}\left[y_{dq} - \frac{\left(y_{dzz} + y_{dzy}\right)}{2}\right] - \frac{1}{2}k_{dw}\left[y_{dw} - \frac{\left(y_{dzz} + y_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dq}\left[\dot{x}_{dq} - \frac{\left(\dot{x}_{dzz} + \dot{x}_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dw}\left[\dot{x}_{dw} - \frac{\left(\dot{x}_{dzz} + \dot{x}_{dzy}\right)}{2}\right] = -m_{dzz}g$$

(26)

$$m_{dzy}\ddot{x}_{dzy} + k_{dzy}x_{dzy} + C_{dzy}\dot{x}_{dzy} + J_{dq}\frac{\ddot{x}_{dzy} - \ddot{x}_{dzz}}{l_d^2} + J_{dw}\frac{\ddot{x}_{dzy} - \ddot{x}_{dzz}}{l_d^2} + J_{dq}\omega\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} + J_{dw}\omega\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} - \frac{1}{2}k_{dq}\left[x_{dq} - \frac{\left(x_{dzz} + x_{dzy}\right)}{2}\right] - \frac{1}{2}k_{dw}\left[x_{dw} - \frac{\left(x_{dzz} + x_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dq}\left[\dot{y}_{dq} - \frac{\left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dw}\left[\dot{y}_{dw} - \frac{\left(\dot{y}_{dzz} + \dot{y}_{dzy}\right)}{2}\right] = -F_{irmx}$$

(27)

$$m_{dzy}\ddot{y}_{dzy} + k_{dzy}y_{dzy} + C_{dzy}\dot{y}_{dzy} + J_{dq}\frac{\ddot{y}_{dzy} - \ddot{y}_{dzz}}{l_d^2} + J_{dw}\frac{\ddot{y}_{dzy} - \ddot{y}_{dzz}}{l_d^2} + J_{dq}\omega\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} + J_{dw}\omega\frac{\dot{y}_{dzy} - \dot{y}_{dzz}}{l_d^2} - \frac{1}{2}k_{dq}\left[y_{dq} - \frac{\left(y_{dzz} + y_{dzy}\right)}{2}\right] - \frac{1}{2}k_{dw}\left[y_{dw} - \frac{\left(y_{dzz} + y_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dq}\left[\dot{x}_{dq} - \frac{\left(\dot{x}_{dzz} + \dot{x}_{dzy}\right)}{2}\right] - \frac{1}{2}C_{dw}\left[\dot{x}_{dw} - \frac{\left(\dot{x}_{dzz} + \dot{x}_{dzy}\right)}{2}\right] = F_{irmy} - m_{dzy}g$$

(28)

3.2.2 Dynamic modelling of a high-pressure rotor system

~ Parameters definition

In figure 4, m_{gzz}, m_{gzy}, m_{gq}, m_{gw} are the equivalent mass of the high-pressure rotor support, the high-pressure compressor, the high-pressure turbine disc and the high-pressure rotor support.

k_{gzz}, k_{gzy}, k_{gq}, k_{gw} are the equivalent stiffness of high pressure rotor support, high pressure compressor, high pressure turbine disc and high pressure rotor support.

C_{gzz}, C_{gzy}, C_{gq}, C_{gw} are equivalent damping of high pressure rotor support, high pressure compressor, high pressure turbine disc and high pressure rotor support.

J_{gpd}, J_{gpp}, J_{gwd}, J_{gwp} are the diameter moment of inertia and polar moment of inertia of the high - pressure compressor and high-pressure turbine disk.

μ_{gq}, μ_{gw} are the eccentricity of high -pressure compressor and high -pressure turbine disc.

ω_g, l_g are the speed and shaft length of the high-pressure rotor.

P_{gqx}, P_{gqy}, P_{gwx}, P_{gwy} are the external forces in X and Y directions of the high pressure compressor and the high pressure turbine disc.

F_{irmx}, F_{irmy} are the reaction forces of the intermediate bearing.

˜ Dynamic modelling of a high-pressure rotor system

The kinetic energy, potential energy and dissipation functions of high -pressure compressor, high -pressure turbine disc and bearing in the Figure 4 are:

$$T_{gq} = \frac{1}{2}m_{gq}\left[\left(\dot{x}_{gq} - u_{gq}\omega_g \sin\omega_g t\right)^2 + \left(\dot{y}_{gq} - u_{gq}\omega_g \cos\omega_g t\right)\right] + \frac{1}{2}\left[J_{gqd}\left(\dot{\theta}_{gx}^2 + \dot{\theta}_{gy}^2\right) + J_{gqd}\omega^2 - 2J_{gqp}\omega\dot{\theta}_{gy}\theta_{gx}\right] \quad (29)$$

$$T_{gw} = \frac{1}{2}m_{gw}\left[\left(\dot{x}_{gw} - u_{gw}\omega_g \sin\omega_g t\right)^2 + \left(\dot{y}_{gw} - u_{gw}\omega_g \cos\omega_g t\right)\right] + \frac{1}{2}\left[J_{gwd}\left(\dot{\theta}_{gx}^2 + \dot{\theta}_{gy}^2\right) + J_{gwd}\omega^2 - 2J_{gwp}\omega\dot{\theta}_{gy}\theta_{gx}\right] \quad (30)$$

$$T_{gz} = \frac{1}{2}m_{gzz}\left(\dot{x}_{gzz}^2 + \dot{y}_{gzz}^2\right) + \frac{1}{2}m_{gzy}\left(\dot{x}_{gzy}^2 + \dot{y}_{gzy}^2\right) \quad (31)$$

$$U_{gq} = \frac{1}{2}k_{gq}\left\{\left(x_{gq} - \left(x_{gzz} + x_{gzy}\right)/2\right)^2 + \left(y_{gq} - \left(y_{gzz} + y_{gzy}\right)/2\right)^2\right\} \quad (32)$$

$$U_{gw} = \frac{1}{2}k_{gw}\left\{\left(x_{gw} - \left(x_{gzz} + x_{gzy}\right)/2\right)^2 + \left(y_{gw} - \left(y_{gzz} + y_{gzy}\right)/2\right)^2\right\} \quad (33)$$

$$U_{gz} = \frac{1}{2}k_{gzz}\left(x_{gzz}^2 + y_{gzz}^2\right) + \frac{1}{2}k_{gzy}\left(x_{gzy}^2 + y_{gzy}^2\right) \quad (34)$$

$$D_{gq} = \frac{1}{2}c_{gq}\left\{\left[\dot{x}_{gq} - \left(\dot{x}_{gzz} + \dot{x}_{gzy}\right)/2\right]^2 + \left[\dot{y}_{gq} - \left(\dot{y}_{gzz} + \dot{y}_{gzy}\right)/2\right]^2\right\} \quad (35)$$

$$D_{gw} = \frac{1}{2}c_{gw}\left\{\left[\dot{x}_{gw} - \left(\dot{x}_{gzz} + \dot{x}_{gzy}\right)/2\right]^2 + \left[\dot{y}_{gw} - \left(\dot{y}_{gzz} + \dot{y}_{gzy}\right)/2\right]^2\right\} \quad (36)$$

$$D_{gz} = \frac{1}{2}c_{gzz}\left(\dot{x}_{gzz}^2 + \dot{y}_{gzz}^2\right) + \frac{1}{2}c_{gzy}\left(\dot{x}_{gzy}^2 + \dot{y}_{gzz}^2\right) \quad (37)$$

Where

$$\theta_{gx} = \frac{y_{gzz} - y_{gzy}}{l_g}, \theta_{gy} = \frac{x_{gzz} - x_{gzy}}{l_g} \quad (38)$$

According to the Lagrange equation of non-conservative systems:

$$\frac{d}{\mathrm{dt}}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = Q_j \quad (39)$$

The differential equation of the high-pressure rotor system is obtained as follows:

$$m_{gq}\ddot{x}_{gq} + k_{gq}\left[x_{gq} - \left(x_{gzz} + x_{gzy}\right)/2\right] + C_{gq}\left[\dot{x}_{gq} - \left(\dot{x}_{gzz} + \dot{x}_{gzy}\right)/2\right] = m_{gq}u_{gq}\omega_g^2 \cos\omega t + P_{gqx} \quad (40)$$

$$m_{gq}\ddot{y}_{gq} + k_{gq}\left[y_{gq} - \left(y_{gzz} + y_{gzy}\right)/2\right] + C_{gq}\left[\dot{y}_{gq} - \left(\dot{y}_{gzz} + \dot{y}_{gzy}\right)/2\right] = m_{gq}u_{gq}\omega_g^2 \sin\omega t + P_{gqy} - m_{gq}\,g \quad (41)$$

$$m_{gw}\ddot{x}_{gw} + k_{gw}\left[x_{gw} - \left(x_{gzz} + x_{gzy}\right)/2\right] + C_{gw}\left[\dot{x}_{gw} - \left(\dot{x}_{gzz} + \dot{x}_{gzy}\right)/2\right] = m_{gw}u_{gw}\omega_g^2 \cos\omega t + P_{gwx} \quad (42)$$

$$m_{gw}\ddot{y}_{gw} + k_{gw}\left[y_{gw} - \left(y_{gzz} + y_{gzy}\right)/2\right] + C_{gw}\left[\dot{y}_{gq} - \left(\dot{y}_{gzz} + \dot{y}_{gzy}\right)/2\right] = m_{gw}u_{gw}\omega_g^2 \sin\omega t + P_{gwy} - m_{gw}g \quad (43)$$

$$m_{gzz}\ddot{x}_{gzz} + k_{gzz}x_{gzz} + C_{gzz}\dot{x}_{gzz} + J_{gq}\frac{\ddot{x}_{gzy} - \ddot{x}_{gzz}}{l_g^2} + J_{gw}\frac{\ddot{x}_{gzy} - \ddot{x}_{gzz}}{l_g^2} - J_{dq}\omega_g\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} - J_{gw}\omega_g\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} - \frac{1}{2}k_{gq}\left[x_{gq} - \frac{\left(x_{gzz} + x_{gzy}\right)}{2}\right] - \frac{1}{2}k_{gw}\left[x_{gw} - \frac{\left(x_{gzz} + x_{gzy}\right)}{2}\right] - \frac{1}{2}C_{gq}\left[\dot{y}_{gq} - \frac{\left(\dot{y}_{gzz} + \dot{y}_{gzy}\right)}{2}\right] - \frac{1}{2}C_{gw}\left[\dot{y}_{gw} - \frac{\left(\dot{y}_{gzz} + \dot{y}_{gzy}\right)}{2}\right] = 0$$

(44)

$$m_{gzz}\ddot{y}_{gzz} + k_{gzz}y_{gzz} + C_{gzz}\dot{y}_{gzz} + J_{gq}\frac{\ddot{y}_{gzy} - \ddot{y}_{gzz}}{l_g^2} + J_{gw}\frac{\ddot{y}_{gzy} - \ddot{y}_{gzz}}{l_g^2} - J_{gq}\omega\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} - J_{gw}\omega\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} - \frac{1}{2}k_{gq}\left[y_{gq} - \frac{(y_{gzz} + y_{gzy})}{2}\right] - \frac{1}{2}k_{gw}\left[y_{gw} - \frac{(y_{gzz} + y_{gzy})}{2}\right] - \frac{1}{2}C_{gq}\left[\dot{x}_{gq} - \frac{(\dot{x}_{gzz} + \dot{x}_{gzy})}{2}\right] - \frac{1}{2}C_{gw}\left[\dot{x}_{gw} - \frac{(\dot{x}_{gzz} + \dot{x}_{gzy})}{2}\right] = -m_{gzz}g$$

(45)

$$m_{gzy}\ddot{x}_{gzy} + k_{gzy}x_{gzy} + C_{gzy}\dot{x}_{gzy} + J_{gq}\frac{\ddot{x}_{gzy} - \ddot{x}_{gzz}}{l_g^2} + J_{gw}\frac{\ddot{x}_{gzy} - \ddot{x}_{gzz}}{l_g^2} + J_{gq}\omega\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} + J_{gw}\omega\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} - \frac{1}{2}k_{gq}\left[x_{gq} - \frac{(x_{gzz} + x_{gzy})}{2}\right] - \frac{1}{2}k_{gw}\left[x_{gw} - \frac{(x_{gzz} + x_{gzy})}{2}\right] - \frac{1}{2}C_{gq}\left[\dot{y}_{gq} - \frac{(\dot{y}_{gzz} + \dot{y}_{gzy})}{2}\right] - \frac{1}{2}C_{gw}\left[\dot{y}_{gw} - \frac{(\dot{y}_{gzz} + \dot{y}_{gzy})}{2}\right] = -F_{irmx}$$

(46)

$$m_{gzy}\ddot{y}_{gzy} + k_{gzy}y_{gzy} + C_{gzy}\dot{y}_{gzy} + J_{gq}\frac{\ddot{y}_{gzy} - \ddot{y}_{gzz}}{l_g^2} + J_{gw}\frac{\ddot{y}_{gzy} - \ddot{y}_{gzz}}{l_g^2} + J_{gq}\omega\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} + J_{gw}\omega\frac{\dot{y}_{gzy} - \dot{y}_{gzz}}{l_g^2} - \frac{1}{2}k_{gq}\left[y_{gq} - \frac{(y_{gzz} + y_{gzy})}{2}\right] - \frac{1}{2}k_{gw}\left[y_{gw} - \frac{(y_{gzz} + y_{gzy})}{2}\right] - \frac{1}{2}C_{gq}\left[\dot{x}_{gq} - \frac{(\dot{x}_{gzz} + \dot{x}_{gzy})}{2}\right] - \frac{1}{2}C_{gw}\left[\dot{x}_{gw} - \frac{(\dot{x}_{gzz} + \dot{x}_{gzy})}{2}\right] = F_{irmy} - m_{gzy}g$$

(47)

IV. DISCUSSION

The dynamic performance of a gas turbine engine is not only related to the structural dimensions, processing precision, and material performance of the rotor system, but also to the working conditions and assembly accuracy. Therefore, in this paper, the methodological content and the composing method for the dynamic modelling of the rotator system, which is an important part of the gas turbine engine, were mentioned. First, a model of a single rotor system with one disk and two supports was conducted. The research and analysis of the vibration response due to the eccentric mass of the rotation disk was conducted, and based on this, the theoretical basis for the dynamic modelling of the dual rotor system was prepared. A mathematical model of the rotor system of the turbofan engine with a double rotor structure was built. A dynamic model was established by dividing a dual rotor system into a low pressure and a high pressure rotor system. Firstly, on the basis of obtaining the equivalent physical quantities of each element of the rotor system, the motion energy equations of low and high pressure compressors, low pressure and high pressure turbine disks, and bearings in the rotor system were established. At this time, the differential equations of the low-pressure and high-pressure rotor systems were established by taking full consideration of the effects of the asymmetry of the low-pressure and high-pressure rotor support, mass eccentricity and the gyroscopic effect, and using the Lagrange equation.

V. CONCLUSION

Unbalance of rotor system is one of the main causes of vibration. In the manufacturing process of the engine, due to uneven material, machining and installation errors, there is inevitably eccentricity, that is, the center of mass deviates from the nominal center of the rotor. In the process of engine work, this mass eccentricity will cause centrifugal force, under the action of centrifugal force, the rotor vibration, through the bearing to the frame, thereby causing the vibration of the whole machine. In this paper, a method of a mathematical modelling of the rotor system of the turbofan engine with a double rotor structure was mentioned. The method of modeling a simple single-rotor system with one disk and two supports was specifically mentioned. A method of a mathematical modelling of the rotor system of the turbofan engine with a double rotor structure was mentioned. In the modelling of the double rotor system, it is divided into two parts, the low-pressure rotor system and the high-pressure rotor system, taking into account the effects of the asymmetry of the support, mass eccentricity, and the gyroscopic effect, and using the Lagrange equation, the differential equation of the high-pressure rotor system and the low-pressure rotor system was created. The created dynamic model provides a theoretical basis for the coefficient design, calculation and analysis of the dynamic performance of the rotor system.

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