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Recursive Computations and Differential and Integral Equations for Summability of Binomial Coefficients with Combinatorial Expressions

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ABSTRACT

This paper presents some findings of recursive computations and its differential and integral calculus for the series and summability of binomial coefficients along with the combinatorial expressions. Nowadays, the growing complexity of mathematical modeling demands the simplicity of mathematical equations for solving today's scientific problems and challenges. In this research article, a model of recursive computations with differential and integral equations is introduced that deals with design and optimization of recursive computations related to the series and summability with real-time functions.

Keywords : Combinatorics, Recursive Computations, Summability, Derivatives, Integral Equations

I. INTRODUCTION

The study of combinatorics, recursive computations, and its differential and integral equations is wide filed in applied mathematics, engineering and medical science, and technology. The numerical technique for solving the sequences and series problems along with its applications [1-12] plays a vital role in mathematical modelling. In this research article a model of recursive computations and its differential and integral equations along with the combinatorial expressions are constituted in order to deal with series and summability. This model can be useful for finding optimized solutions for the problems involving in series and summability and its applications [1-12].

Recursive Computation

In today's technology world it must be understood that the complexity of mathematical modelling demands the simplicity of numerical equations and techniques for solving scientific problems and challenges. In this research article, a model of recursive computations [12] with novel combinatorial expression, and also its differential and integral calculus are constituted for recursive algorithm related to series and summability of binomial coefficients with real-time function(S):

$$S = \sum_{i=k}^{n} V_i^{p+1} f(t)^i \quad (0 \le k \le n)$$

Let us consider x = f(t) and f(t) real-time function. Then, the mathematical expression of recursive computation for summability of binomial coefficients with the combinatorial expression:

$$\sum_{i=0}^{n-1} V_i^{p+1} x^i = \sum_{i=0}^{n-1} V_i^p x^i + \sum_{i=1}^{n-1} V_{i-1}^p x^i + \sum_{i=2}^{n-1} V_{i-2}^p x^i + \cdots + \sum_{i=k}^{n-1} V_{i-k}^p x^i + \cdots + \sum_{i=n-1}^{n-1} V_{i-(n-1)}^p x^i$$

Where V_i^p is a binomial coefficient for the series and summability and its combinatorial formula is:

$$V_i^p = \frac{(i+1)(i+2)(i+3)\dots(i+p)}{p!} \; (1 \le p \le n-1) \& (0 \le i \le n-1).$$

The upper limit of the notation p in the binomial coefficients depends on the maximum limit of the summation of series. For examples,

$$\sum_{i=0}^{n-1} V_i^p x^i \;\&\; (1 \le p \le n-1); \sum_{i=0}^{n} V_i^p x^i \;\&\; (1 \le p \le n); \sum_{i=0}^{n+1} V_i^p x^i \;\&\; (1 \le p \le n+1).$$

The lower limits of the summation of series and the symbol p in the binomial coefficients must originate from 0(Zero) and 1 (One) respectively.

The initial value of the above-mentioned summability equation is:

$$\sum_{i=0}^{n-1} V_i^1 x^i = \frac{nx^{n+1} - (n+1)x^n + 1}{(x-1)^2}, (x \ne 1).$$

In general, the computational model with limits k to $n-1$ is built as

$$\sum_{i=k}^{n-1} V_{i-k}^{p+1} x^i = \sum_{i=k}^{n-1} V_{i-k}^p x^i + \sum_{i=k+1}^{n-1} V_{i-(k+1)}^p x^i + \dots + \sum_{i=n-1}^{n-1} V_{i-(n-1)}^p x^i$$

$$\text{where } V_{i-k}^{p+1} = \frac{(i-k+1)(i-k+2)\dots(i-k+p)(i-k+p+1)}{(p+1)!}$$

The initial values of the equations with limits k to $n-1$ is:

$$\sum_{i=k}^{n-1} V_{i-k}^1 x^i = \frac{(n-k)x^{n+1} - (n-k+1)x^n + x^k}{(x-1)^2}, (\, x \ne 1 \;\&\; 0 \le k \le n-1\,).$$

The iterative computational method shown-above becomes as a real-time system when $x=f(t)$, i.e., function of time.

Calculus for Summability of Binomial Coefficients

Let $y = \sum_{i=0}^{n} x^i$. The derivatives of y are used to form the following equations.

$$\frac{1}{1!}\frac{dy}{dx} = \sum_{i=0}^{n-1} V_i^1 x^i \Leftrightarrow \frac{1}{2!}\frac{d^2y}{dx^2} = \sum_{i=0}^{n-2} V_i^2 x^i \Leftrightarrow \cdots \Leftrightarrow \frac{1}{p!}\frac{d^py}{dx^p} = \sum_{i=0}^{n-p} V_i^p x^i \,.$$

$$\text{Also, } \frac{1}{(p+1)!}\frac{dz}{dx} = \sum_{i=0}^{n-1} V_i^{P+1} x^i \; if \; z = \sum_{i=0}^{n} V_i^p x^i \,.$$

For example, $\frac{1}{2!}\frac{dz}{dx} = \sum_{i=0}^{n-1} V_i^2 x^i \ if \ z = \sum_{i=0}^{n} V_i^1 x^i$.

In general, the derivative for summability of binomial coefficients is expressed as

$$\frac{1}{p!}\frac{d^p w}{dx^p} = \sum_{i=k}^{n-p} V_{i-k}^p x^i \text{ if } w = \sum_{i=k}^{n} x^i.$$

With use of the above recursive computational techniques, the general integral equation below for the summability of binomial coefficients is constituted and it is proved by the basic mathematical knowledge and ideas.

$$(p+1)\int \sum_{i=0}^{n-1} V_i^{p+1} x^i \ dx + C = (p+1)\int \sum_{i=0}^{n-1} V_i^{p+1} x^i \ dx + 1 = \sum_{i=0}^{n} V_i^p x^i \ ,$$

where the constant of the above integral with respect to x is C=1.

Now, the common integral equation is proven using the above recursive computations for summability and its binomial coefficients with the combinatorial expressions.

The mathematical experisons for both $\sum_{i=0}^{n} V_i^p x^i$ and $\sum_{i=0}^{n-1} V_i^{p+1} x^i$ are provied:

$$\sum_{i=0}^{n} V_i^p x^i = 1 + \frac{(p+1)}{1!}x + \frac{(p+1)(p+2)}{2!}x^2 + \cdots + \frac{(n+1)(n+2)\ldots(n+p)}{p!}x^n.$$

$$\sum_{i=0}^{n-1} V_i^{p+1} x^i = 1 + \frac{(p+2)}{1!}x + \frac{(p+2)(p+3)}{2!}x^2 + \cdots + \frac{n(n+1)(n+2)\ldots(n+p)}{(p+1)!}x^{n-1}.$$

By applying integration to the summability $\sum_{i=0}^{n-1} V_i^{p+1} x^i$, it is proved the integral equation.

$$\int \sum_{i=0}^{n-1} V_i^{p+1} x^i \ dx = x + \frac{(p+2)}{1!}\frac{x^2}{2} + \frac{(p+2)(p+3)}{2!}\frac{x^3}{3} + \cdots + \frac{n(n+1)(n+2)\ldots(n+p)}{(p+1)!}\frac{x^n}{n}.$$

$$(p+1)\int \sum_{i=0}^{n-1} V_i^{p+1} x^i \ dx + 1 = 1 + \frac{(p+1)}{1!}x + \frac{(p+1)(p+2)}{2!}x^2 + \frac{(p+1)(p+2)(p+3)}{3!}x^3 + \cdots + \frac{(n+1)(n+2)\ldots(n+p)}{p!}x^n.$$

$$(p+1)\int \sum_{i=0}^{n-1} V_i^{p+1} x^i \ dx + C = (p+1)\int \sum_{i=0}^{n-1} V_i^{p+1} x^i \ dx + 1 = \sum_{i=0}^{n} V_i^p x^i \ .$$

Hence, it is proved.

$$\text{Also,} \qquad \int \sum_{i=0}^{n-1} V_i^1 x^i dx + 1 = \sum_{i=0}^{n} x^i.$$

Examples:

$$2\int \sum_{i=0}^{n-1} V_i^2 x^i \ dx + 1 = \sum_{i=0}^{n} V_i^1 x^i \ ; \ 3\int \sum_{i=0}^{n-1} V_i^3 x^i \ dx + 1 = \sum_{i=0}^{n} V_i^2 x^i;$$

$$4\int \sum_{i=0}^{n-1} V_i^4 x^i \ dx + 1 = \sum_{i=0}^{n} V_i^3 x^i \ ; \ 5\int \sum_{i=0}^{n-1} V_i^5 x^i \ dx + 1 = \sum_{i=0}^{n} V_i^4 x^i; \text{and so on.}$$

In general, the integral equation for summability of binomial coefficients is expressed as

$$(p+1)\int \sum_{i=k}^{n-1} V_{i-k}^{p+1} x^i \ dx + C = \sum_{i=k+1}^{n} V_{i-(k+1)}^{p} x^i + V_{i-k}^{p} x^i = \sum_{i=k}^{n} V_{i-k}^{p} x^i \ ,$$

where the integral constant of this equation $is \ C = V_{i-k}^{p} x^i$.

Note that the maximum limit of the symbol p shown in the binomial coefficients depends on the upper limit of the summation of series and the minimum limits of the summation of series and the notation p mentioned in the binomial coefficients must begin from 0 & 1 respectively.

II. CONCLUSION

The novel techniques of combinatorics, recursive computations, and its differential and integral equations are vital tools used in the fields of engineering and physical sciences and technologies.

In this paper, a model of recursive computations, differential equations, and integral equations have been newly developed that deals with design and optimization of recursive formulae related to series and summability with real-time function

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