# Effect of Cutting Forces on the Form-Shaping Motion In Robotic Milling <br> Ha Thanh Hai <br> ${ }^{1}$ School of Mechanical Engineering, Hanoi University of Science and Technology, University in Hanoi, Vietnam 


#### Abstract

This article presents analysis of dynamics of robots in milling process. A number of factors affects to dynamics model, as well as cutting, form-shaping motion of the robot. Among these factors, cutting forces generating in machining process are strongly variable factor, because of material heterogeneity, depth of cut, machining geometrical surface, etc. Indeed, if cutting force values are constant, the directions of cutting force vectors always change during cutting process. The cutting forces affect to robot's motion, it is hard to determine precisely cutting force values. This paper presents analysis of cutting forces by building computing formulas, deriving and solving differential equations of motion of the robot in milling process. The program of computing and solving the differential equations of motion enables evaluating of the effects of the cutting forces and its calculating errors to form-shaping motion of the robot. The results create a premise to study calibration and elimination effects of cutting force changes with respect to form-shaping motion of robots in milling process.


Keywords: Cutting Force, Robotic Miling, Form-Shaping Motion.

## I. INTRODUCTION

Potentials of applying robot in machining process are huge due to the advantages, to compare with conventional machine tools and CNC machines [1-4]. The most disadvantage of machining using robots is hard to reach to high absolute accuracy. [3-5] presented a number of factors that affect to capability of matching machining accuracy of robots, such as inaccuracy of deriving kinematics and dynamics models, less of structure stiffness, vibration under acting forces, etc. These causes lead to errors of dynamics model and differential equations of motion. This affects to errors in form-shaping motion and machining accuracy. However, the factors that cause errors of kinematic and dynamic parameters, such as lengths of links, masses, center of masses, inertial
tensors, etc., can be eliminated or minimized by calibrating.

The cutting forces applying on the cutter in machining process are unable to be determined precisely. The cutting forces depend on material, depth of cut, cutting speed, etc. When these parameters are constants, the cutting forces values can be evaluated as a constant, but the directions of cutting forces always change due to the complicated geometrical machining surface. In addition, the cutting forces act on the end-effector that is the last link of the kinematic chain, which consists of a large number of joints and links. As a result, computing and expressing the cutting forces in the differential equation of motions are complicated.
[6,7] mentioned analysis of cutting forces, as well as loads on robots in milling processes. [8] introduced a method of trajectory design and analysis of robots in machining. The general method to derive the differential equations of motion of the robots is showed in $[9,10]$. [10] proposed adjusting cutting forces in machine, base on algorithm of inverse kinematic control of robot in milling process. [11-16] presented methods to determine cutting forces in milling process.

The problems of dynamics and controls of robots in machining processes in general, and in high accurate machining such as milling, grinding, etc., to form and shape part surfaces are still challenges so far.

The paper presents deriving of the differential equations of motion of the robot in milling process. The homogeneous transformation matrices are utilized to compute and express the kinematic and dynamic parameters and terms. Expecially, computing the general forces of non-gravitational forces, which mean cutting forces in this paper, are hard. As mentioned above, the cutting forces act on the endeffector of the kinematic chain, which consists of a large number of joints and links, and directions, values of cutting forces vary by time. By using the transformation coordinate matrices, formulating expressions of cutting forces and dynamic parameters and terms in the differential equations of motion are automatic performed by a computer program.
The solution of the differential equations of motion of a specific example is carried out. Suppose that calculating cutting force values exists errors, the results of computing and simulating programs enable to evaluate effect of cutting force errrors to the formshaping motion of the robot. The results provides the basic to conduct next studies, to minimize or elimilate cutting force errors to form-shaping motion of the robot.

The paper consists of 6 parts. After the introduction part, part 2 and part 4 present kinematic and dynamic modelling respectively. Part 4 shows the cutting force model, which bases on the empirical fomulas and calculating cutting forces. Part 5 presents computations and solutions of the dynamic equations of a specific robot milling along a tool path. Assume that manipulating requirements are defined, such as workpiece material, cutter, tool path, cutting process parameters, estimated average values of cutting forces. The solution of dynamic problem is carried out to determine form-shaping motion and driving torques. The numerical computing results enable to evaluate the effects of cutting forces and errors of estimating cutting force values to the form-shaping motion. Part 6 gives conclusion of the study and future works.

## II. ROBOT KINEMATICS

Figure 1 shows presentation of the robot that has a serial structure and 6 degrees of freedom. The method of transformation coordinates and homogeneous transformation matrices are applied to compute and derive kinematic model of the robot.


Figure 1. Kinematics diagram of the machining robot
Table 1 presents notations of the links, frames, homogeneous transformation matrices.

TABLE 1: NOTATION OF LINKS, FRAMES AND DH PARAMETERS OF THE ROBOT

| Link | Frames | Kinematic parameters |  |  |  | Transformation matrix |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\theta_{i}$ | $d_{i}$ | $a_{i}$ | $\alpha_{i}$ |  |
| LK0 | Ooxoyozo | 0 | 0 | 0 | 0 |  |
| LK1 | O1x1y1z1 | $\theta_{1}$ | $d_{1}$ | $a_{1}$ | $\alpha_{1}$ | ${ }^{0} \mathrm{~A}_{1}$ |
| ... | ... |  |  |  |  | $\ldots$ |
| LK6 | O6x6y6z6 | $\theta 6$ | $d 6$ | a6 | $\alpha_{6}$ | ${ }^{5} \mathrm{~A}_{6}$ |
| E | Oexeyeze | $\theta_{E}$ | $d_{E}$ | $a E$ | $\alpha_{E}$ | ${ }^{6} \mathrm{Ae}$ |

Here, LK0 is the fixed link or the fixed base. The frame Ooxoyozo is attached to the fixed base, called the world frame. LK6 is the end-effector, E is the endeffector point, which is attached on the tool frame, called the tool trihedron Oexeyeze, featured by geometric shape of cutting tooth. The position of the tool trihedron Oexeyeze with respect to the operational frame $\mathrm{O}_{6 \times 6 \mathrm{y} 6 \mathrm{z}}$ is determined by constant parameters, that means $\theta_{\mathrm{E}}, \mathrm{d}_{\mathrm{E}}, \mathrm{a}_{\mathrm{E}}, \alpha_{\mathrm{E}}$ are constants due to this two frames are fixed on the end-effector LK6.
The joint variables are notated (1):
$q=\left[q_{1}, . ., q_{6}\right]^{T}=\left[\theta_{1}, . ., \theta_{6}\right]^{T}$
Table 2 shows notations of the clamping table, the clamping frames, the workpiece frames, the workpiece surface trihedrons [8]. In which, the workpiece surface trihedron is determined by geometric shape of workpiece surface, the origin $\mathrm{O}_{\mathrm{i}}$ is on the tool path. The position and orientation of the
 ${ }^{\mathrm{d}} \alpha_{\mathrm{f}},{ }^{,}{ }^{\mathrm{f}} \mathrm{f},{ }^{\mathrm{d}} \eta_{\mathrm{f}}$, notated (2).

## TABLE 2: NOTATION OF LINKS, FRAMES AND DH

 PARAMETERS OF THE CLAMPING TABLE| Link <br> s | Frames | Kinematic parameters |  |  |  |  |  | MT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Xi | yi | Zi | $\alpha_{i}$ | $\beta_{i}$ | $\eta^{\text {i }}$ |  |
| LK0 | Ooxoyozo | 0 | 0 | 0 | 0 | 0 | 0 |  |
| LKB | Obxıybzb | Xb | yb | Zb | $\alpha_{\text {b }}$ | $\beta_{\mathrm{b}}$ | $\eta$ b | ${ }^{0} \mathrm{~A}_{\mathrm{b}}$ |
| LKD | OdXdydzd | Xd | yd | Zd | $\alpha_{\text {d }}$ | $\beta_{\mathrm{d}}$ | $\eta{ }^{\text {d }}$ | ${ }^{\mathrm{b}} \mathrm{Ad}_{d}$ |
| $\mathrm{Ofi}_{\text {fi }}$ | OfiXfiyfizf | $\begin{gathered} \hline{ }^{\mathrm{d}_{\mathrm{X}}} \\ \text { fi } \end{gathered}$ | ${ }^{\text {d }}$ yfi | $\mathrm{d}_{\mathbf{Z f}}$ | $\begin{gathered} \hline{ }^{\mathrm{d}} \alpha \\ \text { fi } \end{gathered}$ | $\bar{\alpha} \beta$ fi | $\begin{aligned} & \hline{ }^{\mathrm{d} \eta} \\ & \\ & \text { fi } \end{aligned}$ | ${ }^{\text {d }} \mathrm{Afi}$ |
| E | Oехеуеz <br> E | $\begin{gathered} \hline{ }^{\mathrm{d}_{\mathrm{X}}} \\ \mathrm{E} \end{gathered}$ | ${ }^{\text {d }} \mathrm{yE}$ | $\begin{gathered} \mathrm{d}_{\mathbf{Z}} \\ \mathrm{E} \end{gathered}$ | $\begin{gathered} \hline{ }^{\mathrm{d}} \boldsymbol{\alpha} \\ \mathrm{E} \end{gathered}$ | $\begin{gathered} \mathrm{d} \beta \\ \mathrm{E} \end{gathered}$ | $\begin{gathered} { }^{\mathrm{d} \eta} \\ \mathrm{E} \end{gathered}$ | ${ }^{\text {d }} \mathrm{Ae}_{\mathrm{E}}$ |

MT is transformation matrice
${ }^{d} p_{f i}=\left[{ }^{d} x_{f i},{ }^{d} y_{f i},{ }^{d} z_{f i},{ }^{d} \alpha_{f i},{ }^{d} \beta_{f i},{ }^{d} \eta_{f i}\right]^{T}$
From parameters (2), position of the workpiece trihedron at a point on the workpiece surface, along the tool path, with respect to the workpiece frame that is expressed by (3).
${ }^{d} A_{f i}\left({ }^{d} p_{f i}\right)=$
$\left[\begin{array}{cc}{ }^{d} C_{f i}\left({ }^{d} \alpha_{f i},{ }^{d} \beta_{f i},{ }^{d} \eta_{f i}\right) & { }^{d} r_{f i}\left({ }^{d} x_{f i},{ }^{d} y_{f i},{ }^{d} z_{f i}\right) \\ 0^{T}\end{array}\right] ;$
The parameters that express the position and orientation of the tool trihedron with respect to the
 (4).
$p={ }^{d} p_{E}=\left[{ }^{d} x_{E},{ }^{d} y_{E},{ }^{d} Z_{E},{ }^{d} \alpha_{E},{ }^{d} \beta_{E},{ }^{d} \eta_{E}\right]^{T}$
The parameters (4) is called operational coordinates. From the parameters (4), the position of the tool trihedron [8] with respect to the workpiece frame is expressed by matrix (5):
${ }^{d} A_{E}\left({ }^{d} p_{E}\right)=$
$\left[\begin{array}{c}{ }^{d} C_{E}\left({ }^{d} \alpha_{E},{ }^{d} \beta_{E},{ }^{d} \eta_{E}\right) \\ 0^{T}\end{array}{ }^{d} r_{E}\left({ }^{d} x_{E},{ }^{d} y_{E},{ }^{d} Z_{E}\right)\right] ;$
The parameters of the Table 2 are constants, because the workpieces is fixed in milling process.
Position and orientation of the tool with respect to the world frame are determined by the two kinematic chains (6), (7):

$$
\begin{gather*}
{ }^{0} A_{E}\left({ }^{d} p_{E}\right)={ }^{0} A_{d}{ }^{d} A_{E}\left({ }^{d} p_{E}\right) \\
=\left[\begin{array}{c}
{ }^{0} C_{E}\left({ }^{d} \alpha_{E},{ }^{d} \beta_{E},{ }^{d} \eta_{E}\right) \\
{ }^{0} r_{E}\left({ }^{d} x_{E},{ }^{d} y_{E},{ }^{d} Z_{E}\right) \\
1
\end{array}\right]  \tag{6}\\
0^{T}
\end{gather*}
$$

From (6), (7), obtained kinematic equation in matrix form (8),
${ }^{0} A_{d}{ }^{d} A_{E}\left({ }^{d} p_{E}\right)={ }^{0} A_{1}\left(q_{1}\right){ }^{1} A_{2}\left(q_{2}\right) \ldots{ }^{5} A_{6}\left(q_{6}\right){ }^{6} A_{E}$

Alternatively, (8) can be expressed by the form (9)
${ }^{d} A_{E}\left({ }^{d} p_{E}\right)={ }^{0} A_{d}^{-1}{ }^{0} A_{1}\left(q_{1}\right){ }^{1} A_{2}\left(q_{2}\right) . .{ }^{5} A_{6}\left(q_{6}\right){ }^{6} A_{E}$
The elements of the left side (9) are functions of the operational coordinate vector $p$ (4). The elements of the right side (9) are functions of the joint coordinate vector q (1).
Direct kinematic computation is performed to determined position, velocity, and acceleration of the
tool with respect to the workpiece, expessed to the workpiece frame. Positions, velocities, and accelerations of the joint variables are able to be measured by sensors. The right side of (9) are determined completely due to the joint variables are determined. Solving equations (9) to find the operational positions, velocities and accelerations of the tool with respect to the workpiece, expessed to the workpiece frame.
From manipulating requirements, motion of the endeffector is determined, inverse kinematic computation is carried out to determine the joint coordinates and its derivative, that means motions of the links are determined to meet the required cutting motion.
From (9), obtained the nonlinear algebraic equations (10).
$f(q, p)=0$
$f=\left[f_{1}, \ldots, f_{6}\right]^{T}$
With the equations (10), in inverse kinematic proplem, the variables are the joint coordinates (1), the parameters varying by time are position of the tool with respect to the machining surface, which is expressed by the operational coordinates $p$ (4).
While conducting milling process, the cutter moves so as to the cutting point of the tooth tracks along the tool path on the machining surface. According to the method of matching trihedrons [8], in the case of form-shaping milling a complicated surface, the workpiece trihedron and the tool trihedron match together to meet the manufacturing demands. Therefore, the operational coordinates (4) are determined by the parameters of the workpiece surface of the tool path ${ }^{d} \mathbf{X f f i}{ }^{\mathrm{d}} \mathbf{y f i},{ }^{\mathrm{d}} \mathbf{Z f i},{ }^{\mathrm{d}} \alpha_{\mathrm{fi}},{ }^{\mathrm{d}} \beta_{\mathrm{fi}},{ }^{\mathrm{d}} \eta_{\mathrm{fi}}$. Solving (10) to computing (1) by (4) can be done by applying the method Newton-Raphson.
According to material removal engineering in implementation, it is required to determine velocity and angular velocity of the tool on the tool path. This enables computing derivatives of the operational coordinates (4), that means (12) and (13) are determined.
$\dot{p}=\left[{ }^{\dot{d}} x_{E}, \dot{d}_{y_{E}}, \dot{d}_{Z_{E}},{ }^{\dot{d}} \alpha_{E},{ }^{\dot{d}} \beta_{E},{ }^{d} \eta_{E}\right]^{T}$
$\ddot{p}=\left[\ddot{d}_{x_{E}}, \ddot{d}_{y_{E}}, \ddot{d}_{Z_{E}}, \ddot{d}_{\alpha_{E}}, \ddot{d}_{\beta_{E}}, \ddot{d} \eta_{E}\right]^{T}$
Carrying out the first derivative of (10) with respect to time t (14):
$J_{q} \dot{q}=J_{p} \dot{p}$
$J_{q}=\frac{\partial f}{\partial q} ; \quad J_{p}=-\frac{\partial f}{\partial p}$
Obtaining the joint velocities, which are expressed by the first derivative of the joint coordinates (16)
$\dot{q}=J_{q}^{-1} J_{p} \dot{p}$
Carrying out the first derivative of (14) with respect to time t (17):
$\dot{J}_{q} \dot{q}+J_{q} \ddot{q}=\dot{J}_{p} \dot{p}+J_{p} \ddot{p}$
Obtaining the joint accelerations, which are expressed by the second derivative of the joint coordinates (18)
$\ddot{q}=J_{q}^{-1}\left[\dot{J}_{p} \dot{p}+J_{p} \ddot{p}-\dot{J}_{q} \dot{q}\right]$

## III.ROBOT's DIFFERENTIAL EQUATION OF MOTION

The paper analysises a serial robot, which have 6 degrees of freedom, employing in milling process to perform form-shaping workpiece surfaces. Figure 1 shows kinematic diagram of the robot that consist of 6 movable links connecting to the fixed base and a clamping table, which clamps the workpieces. The links of robot are notated by LK0 (the fixed link), LK1, LK2, ..., LK6 (the movable links) respectively, in which LK6 is the end-effector. The clamping table is notated by B.
The Lagrange equations of the robot in matrix form can be expressed as follows (19).
$M(q) \ddot{q}+C(q, \dot{q})+G(q)+Q=U$
Here:
$\mathrm{M}(\mathrm{q})$ - is the mass matrix, which is computed as (20). In (20), $m_{i}$ is the mass of link i ; $\mathrm{J}_{\mathrm{Ti}}$ is the Jacobian matrix of the coordinate vector of the center of mass of link $\mathrm{i}^{0}{ }^{\circ}$ ci with respect to the joint coordinates (21); $J_{\mathrm{Ri}}$ is the rotation Jacobian matrix of the angular velocity vector of link i with respect to the derivatives of joint coordinates (22); ${ }^{c i} \Theta_{c i}$ is the inertia tensor of link i about $\mathrm{C}_{\mathrm{i}}$, expressed in the frame which is attached to Ci .
$M(q)=\left[\sum_{\mathrm{i}=1}^{6}\left(J_{\mathrm{Ti}}^{T} m_{i} J_{\mathrm{Ti}}+J_{\mathrm{Ri}}^{T}{ }^{c i} \Theta_{c i} J_{\mathrm{Ri}}\right)\right]_{6 \times 6}$
$J_{T i}=\frac{\partial^{0} r_{c i}}{\partial q}$
$J_{R i}=\frac{\partial^{i} \omega_{i}}{\partial \dot{q}}$
The coordinate vector of the center of mass of link i
${ }^{0}{ }_{\mathrm{rc}}$, the angular velocity vector of link i ${ }^{i} \omega_{i}$ are
computed by transformation matrice from kinematic problem.
$C(q, \dot{q})$ - is the general force vector of coriolis and centrifugal forces (23), (24), (25).
$C(q, \dot{q})=\left[c_{1}, c_{2}, . ., c_{6}\right]^{T}$
$c_{j}=\sum_{k, l=1}^{6}(k, l ; j) \dot{q}_{k} \dot{q}_{l}$
$(k, l ; j)=\frac{1}{2}\left(\frac{\partial m_{k j}}{\partial q_{l}}+\frac{\partial m_{l j}}{\partial q_{k}}-\frac{\partial m_{k l}}{\partial q_{j}}\right)$
With ( $\mathrm{k}, 1 ; \mathrm{j}$ ) is Christofel notation.
$\mathrm{G}(\mathrm{q})$ - is the vector of general forces of gravitational forces (26), (27).
$G(q)=\left[g_{1}, g_{2}, . ., g_{6}\right]^{T}$
$g_{j}=\frac{\partial \Pi}{\partial q_{j}}, \Pi=\sum_{i=1}^{n} \Pi_{i}, \Pi_{i}=m_{i} g z_{c i}$
U - is the vector of general forces of driving forces
(28), (29).
$U=\left[U_{1}, U_{2}, . ., U_{6}\right]^{T}$
$U_{i}=\tau_{i}$
Here, $\tau_{i}$ is the driving force (or torque) of joint $i$ (for prismatic joint, $\tau_{i}$ is force and for revolute joint, $\tau_{i}$ is torque).
$\mathrm{Q}(\mathrm{q})$ - is the vector of general forces and torques of none gravitational forces. The none gravitational forces include acting forces, friction forces, and the elements of cutting forces which cause by the workpiece surface acting on the tool at the contacting point. In this paper, in order to analysis effect of cutting forces, so we only consider the general force Q of the cutting forces.
The cutting force vector that generated by the workpiece surface acting on the tool is notated by $\mathrm{F}_{\mathrm{c}}$, corresponding to general force $\mathrm{Q}(\mathrm{q})$, computed by (30).

$$
\begin{equation*}
Q(q)=J_{F c}^{T} F_{c} \tag{30}
\end{equation*}
$$

Here, $\mathrm{J}_{\mathrm{Fc}}$ is the Jacobian matrix of the vector ${ }^{{ }^{\mathrm{rE}}}{ }^{\mathrm{re}}$, which is the locating vector of the applying point of
the force $\mathrm{F}_{\mathrm{c}}$ with repect to the coordinate vector $\mathrm{q}(1)$. Matrix $\mathrm{J}_{\mathrm{Fc}}$ is computed by (31).

$$
\begin{equation*}
J_{F C}=\frac{\partial^{0} r_{E}}{\partial q} \tag{31}
\end{equation*}
$$

## IV.MILING FORCES



Figure 2. Cutting force expression of the flute i corresponding to an elemental disk dz


Figure 3. Cross-sectional view of an end mill showing differential forces

The cutting forces in milling process are determined by empirical formulas corresponding to specific engineering processes. There are a number of parameters and factors that affect to cutting forces such as: depth of cut t , feed rate s , spindle speed n , width of cut B, etc. From the mechanical model, the milling forces is determined by the model of interactive contaction between the tool and the workpiece surface. Beside that, material removal motion (form-shaping motion) of the cutter also affects to the parameters in the equations that determine cutting forces. So that, corresponding to a certain cutter, the geometric shape of cutter leads to
the model of interactive contaction between the workpiece and the cutter.
The paper analysises the case that an end mill performs a down milling process. The mechanical model that describes interaction between the tool and the workpiece surface, the cutting force elements and engineering parameters, Figure 2 and 3.
Milling forces are generated at the contacting area of the cutter and workpiece. The contacting area has complicated spatial shape, depends on the structure and arrangement of the flutes on the cutter. Consequently, the milling forces are distributed spatial forces, so it is hard to calculate accurately cutting forces. To calculate approximately the cutting force values, the cutter is divided into a number of very thin disks by planes that perpendicular to the axis of the cutter, forming elemental disks that have thickness dz , Figure 2 [11-14]. Milling forces of a elemental disk is formulated, Figure 3, and the resultant forces are integrals of milling forces of the whole depth of cut. According to [11-14], cutting force elements corresponding to the flute j of the elemental disk dz is expressed by (32).
$\left\{\begin{array}{l}d F_{t j}(\theta, z)=\left(K_{t c} a_{j}\left(\theta_{j}(z)\right)+K_{t e}\right) d z \\ d F_{r j}(\theta, z)=\left(K_{r c} a_{j}\left(\theta_{j}(z)\right)+K_{r e}\right) d z \\ d F_{n j}(\theta, z)=\left(K_{n c} a_{j}\left(\theta_{j}(z)\right)+K_{n e}\right) d z\end{array}\right.$
In which:
$\mathrm{dF}_{\mathrm{ij}}, \mathrm{dF}_{\mathrm{r} \mathrm{j}}, \mathrm{dF}_{\mathrm{nj}}$ - are elemental cutting forces of the flute $j$ in tangential, radial and axial directions respectively.
$\mathrm{K}_{\mathrm{tc}}, \mathrm{K}_{\mathrm{rc}}, \mathrm{K}_{\mathrm{nc}}$ - milling force coefficients in tangential, radial and axial directions for linear force model.
$\mathrm{K}_{\mathrm{te}}, \mathrm{K}_{\mathrm{re}}, \mathrm{K}_{\mathrm{ne}}$-Cutting-edge coefficients in tangential, radial and axial directions for the linear force model.
$\mathrm{a}_{\mathrm{j}}(\theta, \mathrm{z})$ - chip thichness of the flute $\mathrm{j}(33)$ :
$a_{j}(\theta, z)=s_{t} \sin \left(\theta_{j}(z)\right)$
$\mathrm{St}_{\mathrm{t}}$ - feed rate: mm/tooth/rev.
$\theta_{\mathrm{j}}(\mathrm{z})$ - the immersion angle changes along the axial direction (34)
$\theta_{j}(z)=\theta(t)+(j-1) \alpha-k_{\beta} z ; \quad \theta(t)=\frac{\pi n}{30} t ; k_{\beta}=$ $\frac{\operatorname{tg} \beta}{R}$
$\alpha$ - the angular between the two consecutive flutes (35).
$\alpha=\frac{2 \pi}{N}$
N - number of flutes of the cutter.
n - cutting tool speed (rmp).
$\beta$ - helix angle of the cutting tool edges.
R - radius of the cutter, D - diameter of the cutter.
From Figure 3, selecting x axis in tangential direction, $y$ axis in perpendicular direction of the tool path at the cutting point, z axis in the axial of the cutter, the elemental forces in $\mathrm{x}, \mathrm{y}, \mathrm{z}$ directions
 (36)
$\left[\begin{array}{l}d F_{x j}\left(\theta_{j}(z)\right) \\ d F_{y j}\left(\theta_{j}(z)\right) \\ d F_{z j}\left(\theta_{j}(z)\right)\end{array}\right]=$
$\left[\begin{array}{ccc}-\cos \theta_{j}(z) & -\sin \theta_{j}(z) & 0 \\ \sin \theta_{j}(z) & -\cos \theta_{j}(z) & 0 \\ 0 & 0 & 1\end{array}\right]\left[\begin{array}{l}d F_{t j}(\theta, z) \\ d F_{r j}(\theta, z) \\ d F_{n j}(\theta, z)\end{array}\right] ;$
Integrating by the elemental forces (36), obtained cutting forces acting on the flute j of cutter (37)
$F_{k j}\left(\theta_{j}(z)\right)=\int_{z_{j 1}}^{z_{j 2}} F_{k j}\left(\theta_{j}(z)\right) d z ; \quad \mathrm{k}=\mathrm{x}, \mathrm{y}, \mathrm{z}$
Here: $\mathrm{z}_{\mathrm{i} 1}\left(\theta_{j}(\mathrm{z})\right), \mathrm{zi}_{\mathrm{j}}\left(\theta_{j}(\mathrm{z})\right)$ are under and upper limitation of the cutting flute $j$.
From (34), obtained $d \theta_{j}(z)=-k_{\beta} d z$, so cutting forces of the flute $j$ (38) [12]

$$
\begin{aligned}
F_{x j}\left(\theta_{j}(z)\right)= & \left\{\frac { s _ { t } } { 4 k _ { \beta } } \left[-\operatorname{K}_{\mathrm{tc}} \cos 2 \theta_{j}(\mathrm{z})+\mathrm{K}_{\mathrm{rc}}\left[2 \theta_{j}(\mathrm{z})\right.\right.\right. \\
& \left.\left.-\sin 2 \theta_{j}(\mathrm{z})\right]\right]+\frac{1}{k_{\beta}}\left[\mathrm{K}_{\mathrm{te}} \sin \theta_{j}(\mathrm{z})-\right. \\
& \left.-\operatorname{K}_{\mathrm{rec}} \cos \theta_{j}(\mathrm{z})\right\} \left\lvert\, \begin{array}{c}
z_{j 2}\left(\theta_{j}(z)\right) \\
z_{j 1}\left(\theta_{j}(z)\right)
\end{array}\right.
\end{aligned}
$$

$$
F_{y j}\left(\theta_{j}(z)\right)=\left\{\frac { - s _ { t } } { 4 k _ { \beta } } \left[\mathrm{K}_{\mathrm{tc}}\left[2 \theta_{j}(\mathrm{z})-\sin 2 \theta_{j}(\mathrm{z})\right]\right.\right.
$$

$$
\left.+\mathrm{K}_{\mathrm{rc}} \cos 2 \theta_{\mathrm{j}}(\mathrm{z})\right]+\frac{1}{k_{\beta}}\left[\mathrm{K}_{\mathrm{te}} \cos \theta_{\mathrm{j}}(\mathrm{z})\right.
$$

$$
\left.\left.-\operatorname{Kr}_{r} \sin \theta_{j}(\mathrm{z})\right]\right\} \left\lvert\, \begin{aligned}
& z_{j_{2}}\left(\theta_{j}(z)\right) \\
& z_{j 1}\left(\theta_{j}(z)\right)
\end{aligned}\right.
$$

$$
\left.F_{z j}\left(\theta_{j}(z)\right)=\left\{\frac{1}{k_{\beta}}\left[\mathrm{Knc}_{\mathrm{n}} \mathrm{~S}_{\mathrm{t}} \cos \theta_{j}(\mathrm{z})-\mathrm{K}_{\mathrm{ne}} \theta_{\mathrm{j}}(\mathrm{z})\right]\right\} \right\rvert\, \begin{gathered}
z_{j 2}\left(\theta_{j}(z)\right) \\
z_{j 1}\left(\theta_{j}(z)\right)
\end{gathered}
$$

The total cutting forces of all flutes at the time that corresponds to the immersion angle $\theta$ are computed by (39)
$F_{x}(\theta)=\sum_{j=1}^{N} F_{x j} ; F_{y}(\theta)=\sum_{j=1}^{N} F_{y j} ; F_{z}(\theta)=$ $\sum_{j=1}^{N} F_{z j}$

## V. NUMERICAL SIMULATIONS



Figure 4. The workpiece and the form - shaping path Numerical computation is carried out with the case of robot perform milling form-shaping of a workpiece surface, shows in Figure 4.

The machining robot model is obtained from ABBrobot IRB 6660.

Geometric and kinematic parameters are presented in Table 3. The other parameters, such as mass, inertial tensor, center of mass, etc, are not presented because of its cumbersome expression.

TABLE 3: KINEMATIC PARAMETERS OF THE ROBOT

| $\mathrm{a}_{1}$ | $\mathrm{~d}_{1}$ | $\mathrm{a}_{2}$ | $\mathrm{a}_{3}$ | $\mathrm{~d}_{4}$ | $\mathrm{~d}_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 300 | 514,5 | 700 | 280 | 1060,24 | 377 |
| ${ }^{6}{ }^{\mathrm{XE}}$ | ${ }^{6} \mathrm{yE}$ | ${ }^{6}{ }^{\mathrm{ZE}}$ | ${ }^{6} \alpha_{\mathrm{E}}$ | ${ }^{6} \beta_{\mathrm{E}}$ | ${ }^{6} \eta_{\mathrm{E}}$ |
| 0 | 0 | 0 | $\pi$ | $\pi / 2$ | 0 |

Units of notation in Table 3: Length - mm, $\mathrm{n}-\mathrm{rmp}$, feed rate $\mathrm{S}_{\mathrm{v}}-\mathrm{mm} /$ tooth/rev.

The workpiece parameters and cutting process parameters are presented in Table 4.

TABLE 4: WORKPIECE, CUTTING PROCESS PARAMETERS AND COEFFICIENTS

|  | Meterials | h | $\mathrm{S}_{\mathrm{v}}$ | n | $\mathrm{K}_{\mathrm{tc}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{Ti}_{6} \mathrm{AL}_{4} \mathrm{~V}$ | 0.5 | 0,1 | 1000 | 1825 |
| 2 | $\mathrm{Ti}_{6} \mathrm{AL}_{4} \mathrm{~V}$ | 0.5 | 0,1 | 1000 | 1698 |
|  | $\mathrm{~K}_{\mathrm{rc}}$ | $\mathrm{K}_{\mathrm{nc}}$ | $\mathrm{K}_{\mathrm{te}}$ | $\mathrm{K}_{\mathrm{re}}$ | $\mathrm{K}_{\text {ne }}$ |
| 1 | 770 | 735 | 29.7 | 55.7 | 1.8 |
| 2 | 438 | 591 | 24.7 | 42.9 | 5.5 |

Using the end-milling cutter that have 12.7 mm diameter, two flute, helix angle $\beta=30^{\circ}$, the start and exit angles are $90^{\circ}$ and $180^{\circ}$ respectively $\theta_{\mathrm{st}}=\pi / 2, \theta_{\mathrm{ex}}=\pi$. Computing data for dynamics problem is conducted following the presented content, for a milling cycle along the tool path L , which is a haft of a circle, radius 40 mm , figure 4
The computing and expressing results of the tool path with respect to the workpiece frame is showed in Figure 5. Figure 6 is the graph of the operational coordinates that expresses the form-shaping motion of robot. Due to the machining plane are planar and the plane of the workpiece frame coincides with the plane of the milling plane, so the operational coordinates in zd direction and rotating about axes xd , $y_{d}$ are equal to zezo


Figure 5. The form-shaping path


Figure 6. Operational coordinates
The joint positions and velocities are computed in the trajectory planning problem, and expressed in Figure 7,8.


Figure 7. Joint coordinates


Figure 8. Joint velocities
Figure 9 depicts part of cutting force graph that calculated for two cases with the cutting coefficients referencing from [16].




Figure 9. Cutting forces in the trihedron frame: redcase 1, blue-case 2

The results of solving dynamic problem are shows in Figure 10. Figure 10 is part of graph that compares the obtained form-shaping motion corresponding to cutting forces showing in Figure 9. The results enables conclusion as follows:


rad

rad

rad

rad


Figure 10. Driving forces: red-withdow vibration, blue-under vibration

Cutting force calculations basing on empirical formulas have remarkable errors, depending on coefficient selections.
With different calculations and parameter selections of cutting forces lead to different approximate results, so that the motion of robot will be existed deviation under effect of cutting forces.
To the next study of employing robot in machining process is control problem. When applying the dynamic model to design controllers for robot, the control forces is computed with cutting force errors, as a result, the obtained motion will have deviations.

## VI.CONCLUSIONS

This paper presents the results of kinematic and dynamic modelling of robots in form-shaping milling process, by utilizing the transformation coordinates and the homogeneous transformation matrices.

The applied algorithms enable computing and programing to derive and solve kinematic and dynamic equations, even robot kinematic structures have a large number of degrees of freedom, and cutting forces calculate and express complicatedly.

The results of solving direct dynamic problem show accuracy of the form-shaping motion, and the milling accuracy is affected by cutting forces.

The study results indicate that it is necessary to find out solutions to guarantee accuracy of form-shaping motion, by calculating precisely cutting forces; calibarating, eliminating or minimizing errors of cutting forces.

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