International Journal of Scientific Research in Mechanical and Materials Engineering
© 2018 IJSRMME | Volume 2 | Issue 2 | ISSN : 2457-0435

A Nature Inspired Algorithm based resolution of an Engineering's ODE

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ABSTRACT

Differential equations have an extremely important task in the engineering field as well as in the ecological, biological and medical fields. They are useful in many domains. By classical point of view, ODEs can be solved simply by usual mathematical tools that are not very accurate especially in the complex problems. Nature Inspired Algorithms are fetching an imperative component of modern optimization when large collections of Nature Inspired Algorithms have appeared recently to treaty successfully a variety of problems. In this paper, we employ the Flower Pollination Algorithm (FPA) proposed by Xin-She Yang (2013), to solve approximately an (IVP) in both linear and nonlinear cases; the efficiency of the planned method is verified by means of a simulation study that shows very good results.

Keywords : Ordinary Differential equations (ODE), Engineering Problems, Initial-Value Problem (IVP); Optimization problem; Flower Pollination Algorithm (FPA).

I. INTRODUCTION

Let $f=f(x,y)$ be a real-valued function of two real variables defined for $a \leq x \leq b$, where a and b are finite, and for all real values of y. The equations

$$\begin{cases} y' = f(x,y) \\ y(a) = y_0 \end{cases} \quad (1)$$

are called first order initial-value problem (IVP); they represent the following problem: To find a function $y(x)$, continuous and differentiable for $x \in [a,b]$ such that $y'=f(x,y)$ from $y(a)=y_0$ for all $x \in [a,b]$ [14]. This problem have unique solution if: f is continuous on $[a,b] \times \mathbb{R}$, and satisfies the Lipschitz condition; it exists a real constant $k>0$, as $|f(x,\theta_1)-f(x,\theta_2)| \leq k|\theta_1 - \theta_2|$, for all $x \in [a,b]$ and all couple $(\theta_1,\theta_2) \in \mathbb{R} \times \mathbb{R}$.

Finding the optimal solutions numerically of +an IVP is gotten with approximations: $y(x_0+h),\ldots,y(x_0+nh)$ where $a=x_0$ and $h=(b-a)/n+1$. For more precision of the solution, we must use a very small step size h that includes a larger number of steps, thus more computing time which is not available in the useful numerical methods like Euler and Runge-Kutta methods [14], that may approximate solutions of IVP and perhaps yield useful information, often sufficing in the absence of exact, analytic solutions.

Biological, physical, or chemical systems in nature are the subject of many nature-inspired metaheuristic algorithms [20]. Swarm Intelligence has become trendy among researchers working on optimization problems all over the world [2] [3] it demonstrates their capabilities in solving many optimization problems by taking a dissimilar forms according to the inspired process of the natural systems like Genetic algorithm [7] and [8], Ant colony optimization algorithm [4], Bee algorithm [10], [12], Particle swarm optimization [9], Bat algorithm [19]...etc. All these algorithms have several advantages illustrated via a wide range of applications.

The Flower Pollination Algorithm (FPA) [18] is a new bio-inspired optimization algorithm that takes off the real life processes of the flower pollination by taking an interesting place between the more recent nature inspired algorithms maintained by its nice performance against several classical metaheuristic algorithms. This is behind the vast utilizations of FPA in various domains such as chemical engineering, civil engineering, electronical and communication engineering, computer science…etc. FPA was hybridized with other nature inspired metaheuristic algorithms in order the overcome its limitations and to benefit from their strength e.g. PSO [20], frog leaping local search [5] and simulated annealing [1], Bat algorithm [9] etc.

The importance of this study is on the methods and techniques for solving IVP are based on FPA. In this paper, IVP is formulated as an optimization problem, and then the FPA [18] is used as a tool to find numerical solutions for this problem.

This paper is organized as follows. The formulation of the problem is revealed in section 2; section 3 provides basics on FPA and its main steps for finding an approximate solution of IVP. The Section 4 exposes an example to show how the FPA can lead to a satisfactory result for solving IVP. The comments and conclusion are made in section 5.

II. PROBLEM FORMULATION

A. Objective function

The main idea in the formulation of the objective function is to use the finite difference formula for the derivative and equation (1) we obtain,

$$\frac{y(x_j) - y(x_{j-1})}{h} \approx f\left(x_{j-1}, y\left(x_{j-1}\right)\right).$$

Thus,

$$\frac{y_j - y_{j-1}}{h} \approx f(x_{j-1}, y_{j-1}).$$

Consequently, we have to consider the error formula:

$$\left[\frac{y_j - y_{j-1}}{h} - f(x_{j-1}, y_{j-1})\right]^2$$

The objective function, associated to Y= (y_1 ,y_2 ,…, y_d) will be:

$$F(y) = \sum_{i=1}^{d}\left[\frac{y_j - y_{j-1}}{h} - f(x_{j-1}, y_{j-1})\right]^2$$
(2)

B. Consistency

We are interested in the calculation of *Y=(*y_1*, y2,…,* y_d *)* which minimizes the objective function in equation (2). We have from Taylor's formula order 1;

$$y_j = y_{j-1} + hy'_{j-1} + O(h^2), \qquad = 1, \ldots, d.$$

So,

$$\frac{y_j - y_{j-1}}{h} = y'_{j-1} + O(h)$$

If we subtract $f(x_{j-1}, y_{j-1})$ from both sides of last equation, we obtain

$$\frac{y_j - y_{j-1}}{h} - f\left(x_{j-1}, y_{j-1}\right) = y'_{j-1} - f\left(x_{j-1}, y_{j-1}\right) + O(h) \quad ,$$
j=1,…,d.

The last relation shows that the final value Y=(y1,y2,…, y_d) is an approximate solution of IVP, for small value of *h*.

III. FLOWER POLLINATION ALGORITHM (FPA)

A. Flower Pollination description

Pollination is very important. It leads to the creation of new seeds that grow into new plants. It begins in the flower. Flowering plants have several different parts that are important in pollination. Flowers have male parts called stamens that produce a sticky powder called pollen. Flowers also have a female part called the pistil. The top of the pistil is called the stigma, and is often sticky. Seeds are made at the base of the pistil, in the ovule. To be pollinated, pollen must be moved from a stamen to the stigma [17]. There are two types of pollination:

1. Self Pollination (Abiotic pollination): Only about 10% of plants fall in this category, it's the fertilization of one flower, when the pollen from a flower pollinates the same flower or flowers of

the same plant; it does not require any pollinators. It occurs when a flower contains both the male and the female gametes, is a process where the pollination happens without involvement of external agents [16] (Figure. 1)

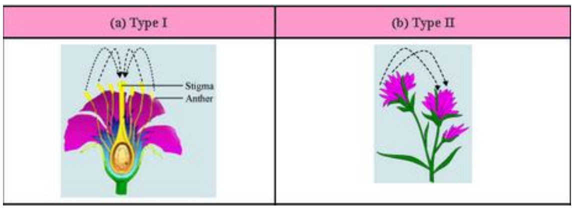

Figure 1. Self pollination.

2. Cross Pollination (biotic pollination): Is typically associated when pollen from a plant's stamen is transferred to a different plant's stigma (of the same species), and such transfer is often linked with pollinators. Pollination occurs in several ways:

People: They can transfer pollen from one flower to another, but most plants are pollinated without any help from people.

Animals: such as bees, butterflies, moths, flies pollinate plants by an accidental way when they are at the plant to get food. The pollinators can fly a long distance, thus they can consider as the global pollination [6]. In addition, bees and birds may behave as Lévy flight behavior [13] [18], with jump or fly distance steps obey a Lévy distribution. Furthermore, flower constancy can be used an increment step using the similarity or difference of two flowers [6] [15].

Wind and Diffusion in water: it picks up pollen from one plant and blows it onto another (Figure. 2)

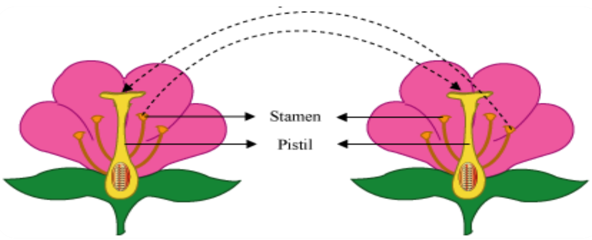

Figure 2. Cross pollination.

B. Formulation of Flower Pollination Algorithm

The four rules given below are used to summarize the above characteristics of pollination process, flower constancy and pollinator behaviour [18].

1. Biotic and cross-pollination is considered as global pollination process and pollinators carrying pollen move in a way that confirms to Levy flights.
2. For local pollination, abiotic pollination and self-pollination are used.
3. Flower constancy can be considered as the reproduction probability is proportional to the similarity of two flowers involved.
4. Local pollination and global pollination is controlled by a switch probability *p∈[0,1].*

To formulate the updating formulas, these rules have to be changed into correct updating equations. The main steps of FPA or simply the flower algorithm are illustrated below:

Pseudo code of the proposed Flower Pollination Algorithm

Objective min or max f(x), x=($\mathbf{x_1}$,x_2 ,..., $\mathbf{x_d}$)
Initialize a population of *n* flowers/pollen gametes with random solutions
Find the best solution $\mathbf{g}_*$ in the initial population
Define a switch probability *p∈[0,1]*
while *(t <MaxGeneration)*
 for *i=1:n* (all *n* flowers in the population)
  if *rand<p*,
  Draw a (*d*-dimensional) step vector *L* which obeys a Levy distribution
  Global pollination via $x_i^{t+1} = x_i^t + L(\mathbf{g}_* - x_i^t)$
  else
  Draw from a uniform distribution in *[0,1]*
  Randomly choose *j* and *k* among all the solutions
  Do local pollination via $x_i^{t+1} = x_i^t + \epsilon(x_j^t - x_k^t)$
  end if
   Evaluate new solutions
   If new solutions are better, update them in the population
 end for
 Find the current best solution $\mathbf{g}_*$
end while

In principle, flower pollination process can happen at both local and global levels. But in reality, flowers in the neighborhood have higher chances of getting pollinated by pollen from local flowers than those which are far away. To simulate this feature, a proximity probability (Rule 4) can be commendably used to switch between intensive local pollination to common global pollination. To start with, a raw value of *p=0.5* may be used as an initial value. A preliminary parametric study indicated that *p=0.8* may work better for most applications

IV. NUMERICAL RESULTS

During our experimental study, the FPA and its operators are coded in MATLAB (R 2013a) for the solution of IVP. The numerical results are shown in graphical and tabular form.

The problem treatment demanding, two types of parameter, the first are related to FPA and the second are related to IVP. These parameters are described as follows:

1. **FPA related parameter:** In this work, the parameters adopted by the FPA in each problem are summarized in the following table:

Table1. Parameters adopted by the FPA

Parameter	Quantity
Dimension of the search variables (d)	10
Total number of iterations (N)	2000
Population size (n)	20
Probability switch (p)	0.8

2. **IVP related parameter**: FPA is an optimization instrument. Then, the essential differential equation is converting into discretization form. The backward difference formula is used to convert differential equation into discretization form when the derivative term is replaced in the discretized form by a difference quotient for approximations.

1. The interval of the IVP is equally partitioned into *n+* 1 equidistant subinterval with the length *h=(b-a)/n+1*. Where *n* = 9 is a number of interior nodes. The IVP related parameters are as follows:

1. the number of interior nodes (*n* = 9).
2. The step size *h* = 0.1, h=0.2 for case1 and case2 respectively.
3. The initial condition and the interval between which the differential equation is solved are varying from case to case.

Now we proceed to the experimental study when we introduce two types of IVP that are the linear and nonlinear cases:

The objective function

$$F(y_1, y_2, \dots, y_{10}) = \sum_{i=1}^{10}\left[\frac{y_j - y_{j-1}}{h} - f(x_{j-1}, y_{j-1})\right]^2$$
$$= \sum_{i=1}^{10}\left[\frac{y_j - y_{j-1}}{h} - y_{j-1}\right]^2$$

Case1: (linear first order IVP) Let us look at a simple (IVP):

$$\begin{cases} \dfrac{dy}{dx} = y, \qquad 0 \le x \le 1 \\ y(0) = 1 \end{cases}$$

For *d*=10, *h=1-0/10=0.1,* $x_0 = 0$, $y_0 = 1$, The exact solution is *y(x)= exp(x).*

The obtained results, the comparison between the exact solution and the FPA results with absolute error values are shown in Table 2.

Table2. Exact and Numerical results by FPA method of case1 example for d=10

i	x_i	Exact Results	FPA Results	Absolute Error
1	0.1000	1.1052	1.1053	0.0001
2	0.2000	1.2214	1.2215	0.0001
3	0.3000	1.3499	1.3492	0.0007
4	0.4000	1.4918	1.4907	0.0011
5	0.5000	1.6487	1.6467	0.0020
6	0.6000	1.8221	1.8191	0.0030
7	0.7000	2.0138	2.0093	0.0045
8	0.8000	2.2255	2.2195	0.0060
9	0.9000	2.4596	2.4524	0.0072
10	1.000	2.7183	2.7094	0.0089

Numerical results in Table 2 are illustrated in the Figure 3.

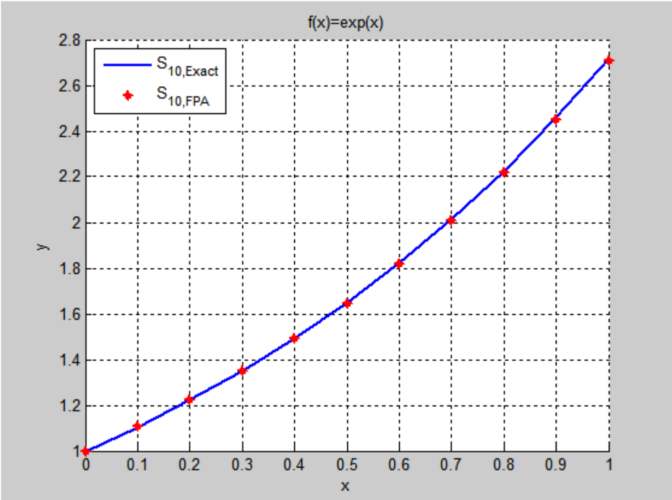

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Figure 3. Numerical solutions plot of case1 example for d=10.

Case 2: (Nonlinear first order IVP) We are now going to start looking at nonlinear first order separable IVP.

$$\begin{cases} \frac{dy}{dx} = 6y^2x, \qquad 1 \le x \le 3 \\ y(1) = \frac{1}{25}. \end{cases}$$

For *d*=10, *h=3-1/10=0.2*, $x_0 = 1$, $y_0 = 1/25$, The exact solution is $y(x)=\frac{1}{1/28-3x^2}$

The achieved results, the comparison among the precise solution and the FPA outcomes with absolute error values are shown in Table 3.

Table3. Exact And Numerical Results By FPA Method of case 2 For d=10

i	x_i	Exact Results	FPA Results	Absolute Error
1	1.0000	0.0400	0.0380	0.0020
2	1.2000	0.0422	0.0419	0.0003
3	1.4000	0.0452	0.0449	0.0003
4	1.6000	0.0492	0.0488	0.0004
5	1.8000	0.0547	0.0545	0.0002
6	2.0000	0.0625	0.0620	0.0005
7	2.2000	0.0742	0.0738	0.0004
8	2.4000	0.0933	0.0929	0.0004
9	2.6000	0.1295	0.1290	0.0005
10	2.8000	0.2232	0.2229	0.0003
11	3.0000	1.0000	0.9999	0.0001

Numerical outcomes in Table 3 are showed via the Figure 4.

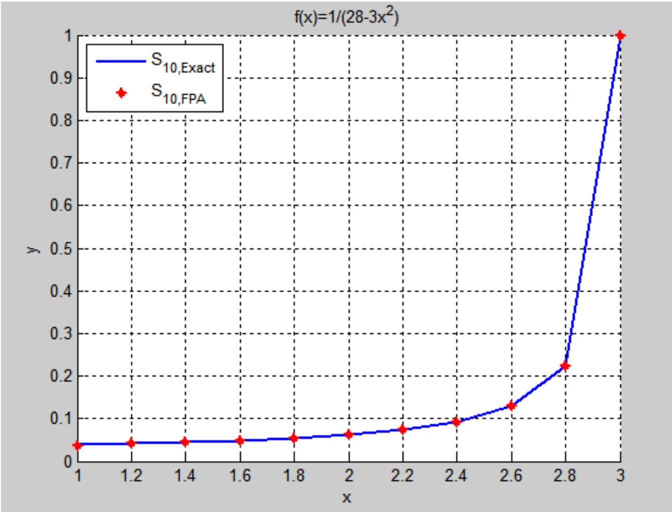

Figure 4. Numerical solutions plot of case2 example for d=10

Discussions: The new FPA have successfully developed to take off the characteristics of flower pollination. Our simulation results indicate that FPA is simple, reduces time, flexible and exponentially better to solve optimization IVP.

V. CONCLUSION

In this study, we apply the FPA to solve approximately an (IVP) for both linear and nonlinear cases, after a comparison between the exact solutions and the algorithm outcomes of the chosen examples, the results found are very adequately precise of the solution in its preliminary formulation.

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